

CHAPTER-2

RATIONAL NUMBERS

Students Learning Outcomes

After studying this chapter, students will be able to:

- Define a rational number as a number that can be expressed in the form, where p and q are integers and $q \neq 0$.
- Represent rational numbers on the number line.
- Add two or more rational numbers.
- Subtract a rational number from another.
- Find additive inverse of rational numbers.
- Multiply two or more rational numbers.
- Divide a rational number by a non-zero rational number.
- Find multiplicative inverse of a rational number.
- Find a reciprocal of a rational number.
- Verify commutative property of rational numbers with respect to addition and multiplication.
- Verify associative property of rational numbers with respect to addition and multiplication.
- Verify distributive property of rational numbers with respect to multiplication over addition/subtraction.
- Compare two rational numbers.
- Arrange rational numbers in ascending or descending order.

SOLVED EXERCISE 2.1

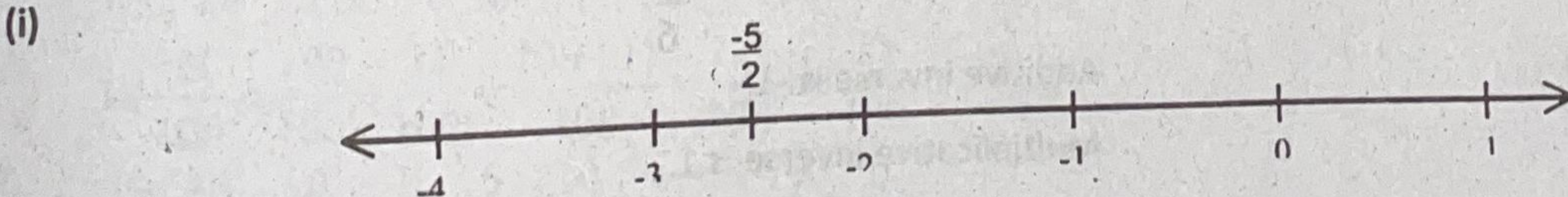
1. Write "T" for a true and "F" for a false statement.

- (i) Positive numbers are rational numbers.
- (ii) "0" is not a rational number.
- (iii) An integer is expressed in form.
- (iv) Negative numbers are not rational numbers.
- (v) In any rational number, q can be zero.

Answers:

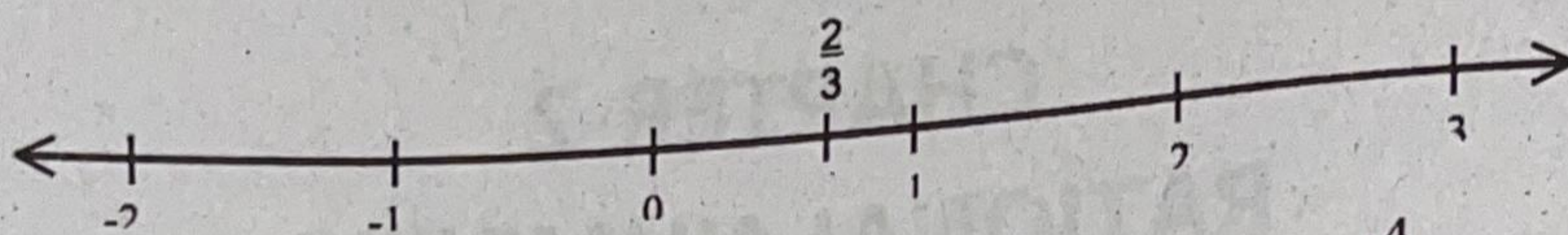
(i)	T	(ii)	F	(iii)	F	(iv)	F	(v)	F
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2. Represents each rational number on the number line.

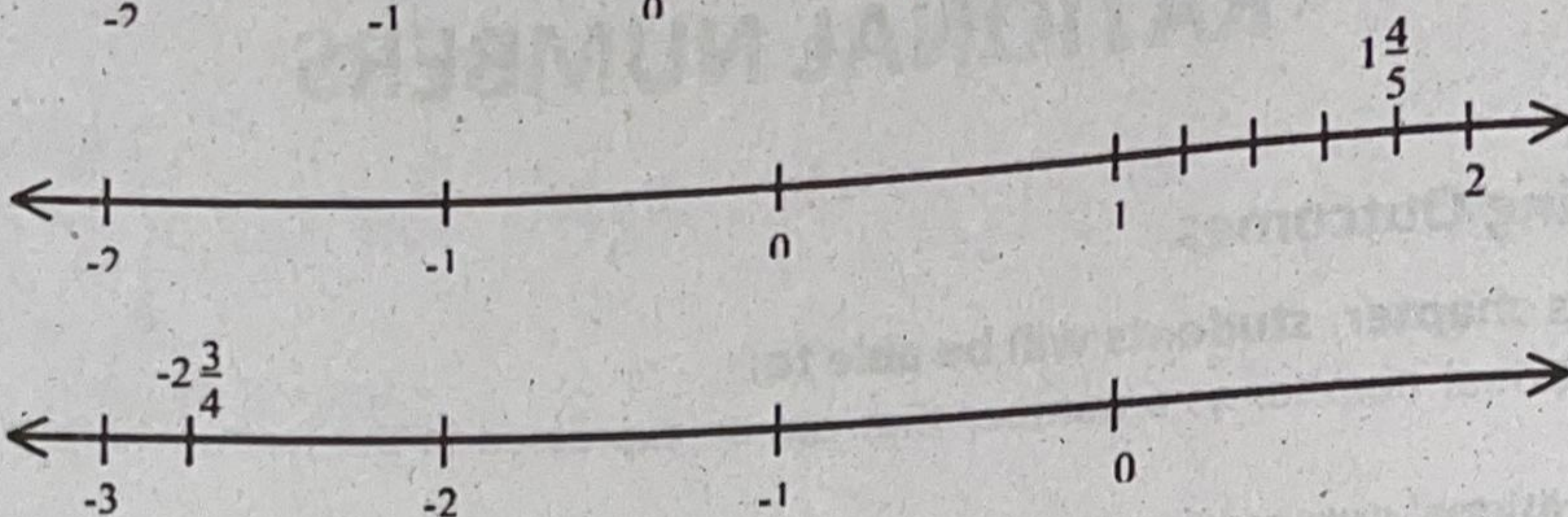


(ii)

(iii) 1



(iv) -2



SOLVED EXERCISE 2.2

1. Find the additive inverse and multiplicative inverse of the following rational numbers.

(i) $(-7) + 7 = 0$

Additive inverse is 7.

$$(-7) \times \left(-\frac{1}{7}\right) = 1$$

Multiplicative inverse is $-\frac{1}{7}$.

(ii) $23 + (-23) = 0$

Additive inverse is -23

$$23 \times (1/23) = 1$$

Multiplicative inverse is $1/23$

(iii) $(-11) + 11 = 0$

Additive inverse is 11

$$(-11) \times \left(-\frac{1}{11}\right) = 1$$

Multiplicative inverse is $-\frac{1}{11}$.

(iv) $\frac{1}{3} + \left(-\frac{1}{3}\right) = 0$

Additive inverse is $-\frac{1}{3}$.

$$\left(\frac{1}{3}\right) \times 3 = 1$$

Multiplicative inverse is 3.

(v) $\frac{-2}{7} + \left(\frac{2}{7}\right) = 0$

Additive inverse is $\frac{2}{7}$.

$$\left(\frac{-2}{7}\right) \times \left(\frac{-7}{2}\right) = 1$$

Multiplicative inverse is $\frac{-7}{2}$.

(vi) $6 + (-6) = 0$

Additive inverse is -6.

$$6 \times \frac{1}{6} = 1$$

Multiplicative inverse is $\frac{1}{6}$.

(vii) $1 + (-1) = 0$

Additive inverse is -1.

$$1 \times (1) = 1$$

Multiplicative inverse is 1.

(viii) $\left(\frac{-6}{13}\right) + \frac{6}{13} = 0$

Additive inverse is $\frac{6}{13}$.

Multiplicative inverse is $\frac{-13}{6}$.

Additive inverse is $\frac{-1}{100}$.

Multiplicative inverse is 100.

Additive inverse is $\frac{-18}{27}$.

Multiplicative inverse is $\frac{27}{18}$.

Additive inverse is $\frac{99}{100}$.

Multiplicative inverse is $\frac{-100}{99}$.

Additive inverse is $-\frac{102}{117}$.

Multiplicative inverse is $\frac{117}{102}$.

$$\left(\frac{-6}{13}\right) \times \left(\frac{-13}{6}\right) = 1$$

$$(ix) \quad \left(\frac{1}{100}\right) + \left(\frac{-1}{100}\right) = 0$$

$$\left(\frac{1}{100}\right) \times \left(\frac{100}{1}\right) = 1$$

$$(x) \quad \frac{18}{27} + \left(\frac{-18}{27}\right) = 0$$

$$\frac{18}{27} \times \frac{27}{18} = 1$$

$$(xi) \quad \left(-\frac{99}{100}\right) + \frac{99}{100} = 0$$

$$-\frac{99}{100} \times \frac{-100}{99} = 1$$

$$(xii) \quad \frac{102}{117} + \left(-\frac{102}{117}\right) = 0$$

$$\frac{102}{117} \times \frac{117}{102} = 1$$

2. Simplify the following:

Solution:

$$(i) \quad \frac{1}{8} - \left(-\frac{5}{8}\right) = \frac{1}{8} + \frac{5}{8} = \frac{6}{8} = \frac{3}{4}$$

$$(ii) \quad -\frac{99}{100} + \frac{77}{100} = \frac{-99+77}{100} = \frac{-22}{100} = \frac{-11}{50}$$

$$(iii) \quad \frac{3}{4} + \frac{4}{3} = \frac{9}{12} + \frac{16}{12} = \frac{9+16}{12} = \frac{25}{12} = 2\frac{1}{12}$$

$$(iv) \quad \frac{1}{5} - \frac{3}{20} = \frac{4}{20} - \frac{3}{20} = \frac{4-3}{20} = \frac{1}{20}$$

$$(v) \quad 1 + \left(\frac{-49}{50}\right) = \frac{50}{50} + \left(\frac{-49}{50}\right) = \frac{50-49}{50} = \frac{1}{50}$$

$$(vi) \quad 1 + \frac{11}{100} = \frac{100}{100} + \frac{11}{100} = \frac{100+11}{100} = \frac{111}{100} = 1\frac{11}{100}$$

$$(vii) \quad \frac{1}{11} + \frac{-5}{11} + \frac{10}{11} = \frac{1}{11} - \frac{5}{11} + \frac{10}{11} = \frac{1-5+10}{11} = \frac{6}{11}$$

$$(viii) \quad \frac{13}{23} - \frac{10}{23} + \frac{4}{23} = \frac{13-10+4}{23} = \frac{7}{23}$$

$$(ix) \quad -\frac{1}{2} + \left(-\frac{1}{5}\right) + \frac{9}{10} = -\frac{5}{10} - \frac{2}{10} + \frac{9}{10} = \frac{-5-2+9}{10} = \frac{2}{10} = \frac{1}{5}$$

$$(x) \quad \frac{1}{8} + \frac{1}{9} - \frac{15}{18} = \frac{9}{72} + \frac{8}{72} - \frac{60}{72} = \frac{9+8-60}{72} = \frac{-43}{72}$$

$$(xi) \quad -\frac{3}{4} - \frac{5}{6} - \left(-\frac{17}{8}\right) = -\frac{3}{4} - \frac{5}{6} + \frac{17}{8} = -\frac{18}{24} - \frac{20}{24} + \frac{51}{24} = \frac{-18-20+51}{24} = \frac{13}{24}$$

$$(xii) \quad \frac{1}{11} + \frac{11}{10} + \left(-\frac{22}{5}\right) = \frac{10}{110} + \frac{121}{110} - \frac{484}{110} = \frac{10+121-484}{110} = \frac{131-484}{110} = -\frac{353}{110} = -3\frac{23}{110}$$

3. Simplify:

Solution:

$$(i) \quad \frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$$

$$(ii) \quad \frac{50}{51} \times \frac{7}{10} = \frac{35}{51}$$

$$(iii) \quad \frac{121}{169} \div \frac{11}{13} = \frac{121}{169} \times \frac{13}{11} = \frac{11}{13}$$

$$(iv) \quad \frac{5}{7} \div \frac{35}{40} = \frac{5}{7} \times \frac{40}{35} = \frac{40}{49}$$

$$(v) \quad -\frac{15}{28} \times \frac{14}{30} = -\frac{1}{4}$$

$$(vi) \quad \frac{111}{100} \div \frac{222}{300} = \frac{111}{100} \times \frac{300}{222} = \frac{3}{2} = 1\frac{1}{2}$$

$$(vii) \quad \frac{3}{2} \div \frac{4}{9} \times \frac{16}{81} = \frac{3}{2} \times \frac{9}{4} \times \frac{16}{81} = \frac{2}{3}$$

$$(viii) \quad \frac{8}{9} \div \frac{2}{3} \times \frac{15}{28} = \frac{8}{9} \times \frac{3}{2} \times \frac{15}{28} = \frac{15}{21} = \frac{5}{7}$$

$$(ix) \quad \frac{8}{125} \div \frac{16}{75} = \frac{8}{125} \times \frac{75}{16} = \frac{3}{10}$$

$$(x) \quad \frac{1}{5} \times \left(-\frac{2}{5}\right) \times \left(-\frac{100}{32}\right) = \frac{1}{4}$$

$$(xi) \quad \frac{1}{1000} \div -\frac{1}{100} = \frac{1}{1000} \times \left(-\frac{100}{1}\right) = -\frac{1}{10}$$

$$(xii) \quad -\frac{1}{2} \times \frac{3}{5} \div \left(-\frac{51}{40}\right) = -\frac{1}{2} \times \frac{3}{5} \times -\frac{40}{51} = \frac{4}{17}$$

SOLVED EXERCISE 2.3

1. Put the correct sign $>$, $<$ or $=$ between the following pairs of rational numbers.

Solution:

$$(i) \frac{1}{2} < \frac{15}{20}$$

$$(ii) \frac{2}{-3} < \frac{1}{6}$$

$$(iii) \frac{-1}{5} = \frac{-2}{10}$$

$$(iv) \frac{-1}{9} > \frac{-4}{3}$$

$$(v) -1 < \frac{-2}{3}$$

$$(vi) \frac{1}{2} < 1$$

$$(vii) \frac{5}{7} > \frac{-1}{2}$$

$$(viii) \frac{11}{-10} < \frac{-10}{11}$$

$$(ix) \frac{4}{-100} = \frac{-1}{25}$$

$$(x) \frac{-4}{7} > \frac{-5}{2}$$

$$(xi) \frac{4}{-100} = \frac{-1}{25}$$

$$(xii) \frac{-8}{11} < \frac{3}{-10}$$

2. Arrange the following rational numbers in descending order.

Solution:

$$(i) \frac{8}{9}, \frac{2}{3}, \frac{1}{2}$$

$$(ii) \frac{3}{4}, \frac{1}{2}, \frac{1}{6}$$

$$(iii) \frac{5}{6}, \frac{4}{7}, \frac{1}{3}$$

3. Arrange the following rational numbers in ascending order.

Solution:

$$(i) \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

$$(ii) \frac{1}{10}, \frac{2}{15}, \frac{4}{5}$$

$$(iii) \frac{1}{4}, \frac{3}{8}, \frac{5}{6}$$

4. Prove that:

Solution:

$$(i) \left(\frac{-1}{2} \right) + \frac{1}{3} = \frac{1}{3} + \left(\frac{-1}{2} \right)$$

$$\frac{-3+2}{6} = \frac{2-3}{6}$$

$$\frac{-1}{6} = \frac{-1}{6}$$

$$(iii) \left(\frac{12}{-105} \right) \times \left(\frac{-15}{84} \right) = \frac{-15}{84} \times \frac{-12}{105}$$

$$\frac{4}{35} \times \frac{5}{28} = \frac{5}{28} \times \frac{4}{35}$$

$$\frac{1}{49} = \frac{1}{49}$$

$$(ii) \frac{10}{11} + \left(\frac{5}{-44} \right) = \left(\frac{5}{-44} \right) + \frac{10}{11}$$

$$\frac{40-5}{44} = \frac{-5+40}{44}$$

$$\frac{35}{44} = \frac{35}{44}$$

$$(iv) -\frac{2}{3} \times \left(\frac{7}{8} \times \frac{9}{14} \right) = \left(-\frac{2}{3} \times \frac{7}{8} \right) \times \frac{9}{14}$$

$$-\frac{2}{3} \times \frac{9}{16} = \frac{-7}{12} \times \frac{9}{14}$$

$$(v) \quad \frac{3}{5} + \left(\frac{1}{2} + \frac{7}{10} \right) = \left(\frac{3}{5} + \frac{1}{2} \right) + \frac{7}{10}$$

$$\frac{3}{5} + \frac{5+7}{10} = \frac{6+5}{10} + \frac{7}{10}$$

$$\frac{3}{5} + \frac{12}{10} = \frac{11}{10} + \frac{7}{10}$$

$$\frac{6+12}{10} = \frac{11+7}{10}$$

$$\frac{18}{10} = \frac{18}{10}$$

$$\frac{9}{5} = \frac{9}{5}$$

$$(vii) \quad \frac{2}{3} \times \left(\frac{1}{2} + \frac{5}{6} \right) = \left(\frac{2}{3} \times \frac{1}{2} \right) + \left(\frac{2}{3} \times \frac{5}{6} \right)$$

$$\frac{2}{3} \times \frac{3+5}{6} = \frac{1}{3} + \frac{5}{9}$$

$$\frac{2}{3} \times \frac{8}{6} = \frac{3+5}{9}$$

$$\frac{8}{9} = \frac{8}{9}$$

$$(ix) \quad \frac{-5}{8} \times \left(\frac{4}{7} - \frac{2}{3} \right) = \left(\frac{-5}{8} \times \frac{4}{7} \right) - \left(\frac{-5}{8} \times \frac{2}{3} \right)$$

$$\frac{-5}{8} \times \frac{12-14}{21} = \frac{-5}{14} + \frac{5}{12}$$

$$\frac{-5}{8} \times \frac{-2}{21} = \frac{-30+35}{84}$$

$$\frac{5}{84} = \frac{5}{84}$$

$$-\frac{3}{8} = \frac{-3}{8}$$

$$(vi) \quad -\frac{1}{2} + \left(\frac{3}{5} + \frac{1}{4} \right) = \left(\frac{1}{-2} + \frac{3}{5} \right) + \frac{1}{4}$$

$$-\frac{1}{2} + \frac{12+5}{20} = \frac{-5+6}{10} + \frac{1}{4}$$

$$-\frac{1}{2} + \frac{17}{20} = \frac{1}{10} + \frac{1}{4}$$

$$\frac{-10+17}{20} = \frac{2+5}{20}$$

$$\frac{7}{20} = \frac{7}{20}$$

$$(viii) \quad \frac{1}{4} \times \left(\frac{8}{9} - \frac{12}{15} \right) = \left(\frac{1}{4} \times \frac{8}{9} \right) - \left(\frac{1}{4} \times \frac{12}{15} \right)$$

$$\frac{1}{4} \times \frac{40-36}{45} = \frac{2}{9} - \frac{1}{5}$$

$$\frac{1}{4} \times \frac{4}{45} = \frac{10-9}{45}$$

$$\frac{1}{45} = \frac{1}{45}$$

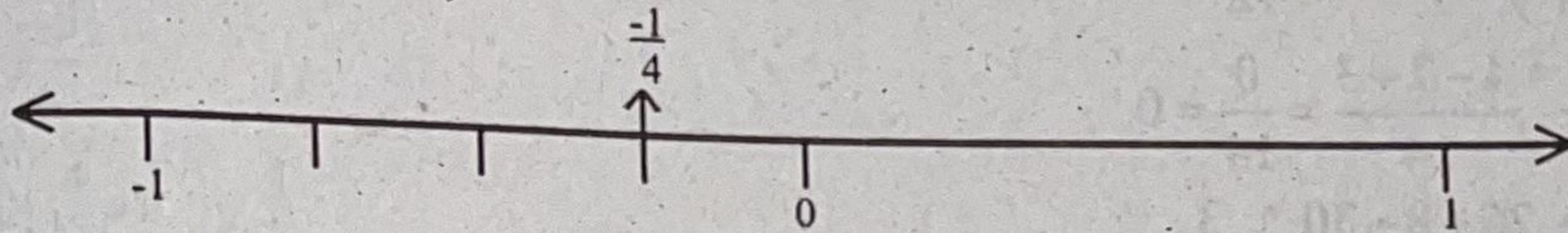
$$(x) \quad \frac{24}{49} \times \left(\frac{7}{8} + \frac{14}{6} \right) = \left(\frac{24}{49} \times \frac{7}{8} \right) + \left(\frac{24}{49} \times \frac{14}{6} \right)$$

$$\frac{24}{49} \times \frac{21+56}{24} = \frac{3}{7} + \frac{8}{7}$$

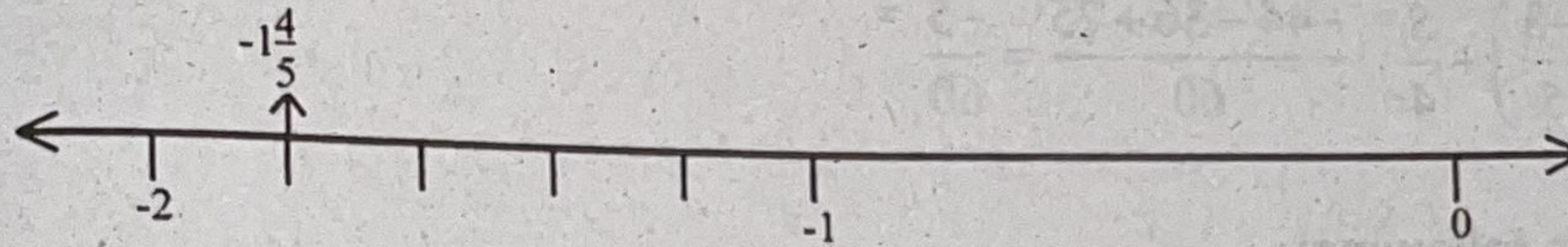
$$\frac{24}{49} \times \frac{77}{24} = \frac{3+8}{7}$$

$$\frac{11}{7} = \frac{11}{7}$$

(iii) $-\frac{1}{4}$



(iv) $-1\frac{4}{5}$



5. Find the additive and multiplicative inverse of the following rational numbers.

(i) $-14 + 14 = 0$

Additive inverse is 14.

$(-14) \times \left(-\frac{1}{14}\right) = 1$

Multiplicative inverse is $-\frac{1}{14}$.

(ii) $(1/5) + \left(-\frac{1}{5}\right) = 0$

Additive inverse is $-\frac{1}{5}$.

$(1/5) \times 5 = 1$

Multiplicative inverse is 5.

(iii) $-2 + 2 = 0$

Additive inverse is 2.

$-2 \times -1/2 = 1$

Multiplicative inverse is $-1/2$.

(iv) $6 + (-6) = 0$

Additive inverse is -6.

$6 \times (1/6) = 1$

Multiplicative inverse is $1/6$.

6. Put the correct sign $>$ or $<$ between the following pairs of rational numbers.

(i) $\frac{1}{4} > -\frac{1}{2}$

(ii) $\frac{2}{3} > \frac{1}{5}$

(iii) $-\frac{11}{17} < \frac{3}{8}$

(iv) $\frac{10}{13} < \frac{11}{14}$

(v) $-\frac{4}{9} < \frac{2}{-5}$

(vi) $\frac{5}{-22} > \frac{-11}{25}$

7. Solve the following:

Solution:

(i) $\left(\frac{-19}{55}\right) + \frac{51}{55} + \left(\frac{-21}{55}\right) = \frac{-19+51-21}{55} = \frac{11}{55} = \frac{1}{5}$

(ii) $\frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$

$$(iii) \quad \left(\frac{-1}{3}\right) + \left(\frac{-1}{4}\right) + \frac{1}{2} = \frac{-4-3+6}{12} =$$

$$(iv) \quad \frac{2}{7} - \frac{1}{2} + \frac{3}{14} = \frac{4-7+3}{14} = \frac{0}{14} = 0$$

$$(v) \quad \frac{5}{8} + \frac{1}{5} - \frac{3}{4} = \frac{25+8-30}{40} = \frac{3}{40}$$

$$(vi) \quad \left(\frac{-11}{15}\right) + \left(\frac{-3}{5}\right) + \frac{5}{4} = \frac{-44-36+75}{60} = \frac{-5}{60} =$$

8. Simplify the following:

Solution:

$$(i) \quad \frac{2}{3} \div \frac{16}{21} \times \frac{27}{49} = \frac{2}{3} \times \frac{21}{16} \times \frac{27}{49} = \frac{27}{56}$$

$$(ii) \quad \left(\frac{-1}{100}\right) \div \left(\frac{1}{10}\right) = \frac{-1}{100} \times \frac{10}{1} = \frac{-1}{10}$$

$$(iii) \quad \frac{1}{5} \times \frac{2}{3} \times \left(\frac{-30}{44}\right) = \frac{1}{5} \times \frac{-10}{22} = \frac{-1}{11}$$

$$(iv) \quad \frac{1}{6} \times \left(\frac{-2}{3}\right) \div \left(\frac{-11}{63}\right) = \frac{1}{6} \times \frac{-2}{3} \times \frac{-63}{11} = \frac{-1}{9} \times \frac{-63}{11} = \frac{7}{11}$$

$$(v) \quad \frac{-2}{3} \div \frac{3}{4} \times \frac{63}{100} = \frac{-2}{3} \times \frac{4}{3} \times \frac{63}{100} = \frac{-8}{9} \times \frac{63}{100} = \frac{-14}{25}$$

$$(vi) \quad \frac{8}{21} \div \frac{7}{12} = \frac{8}{21} \times \frac{12}{7} = \frac{32}{49}$$

9. Prove that:

Solution:

$$(i) \quad (-1) + \frac{35}{34} = \frac{35}{34} + (-1)$$

$$\frac{-34+35}{34} = \frac{35-34}{34}$$

$$\frac{1}{34} = \frac{1}{34}$$

$$(ii) \quad \frac{-4}{5} \times \left(\frac{1}{8} + \frac{11}{12}\right) = \left(\frac{-4}{5} \times \frac{1}{8}\right) + \left(\frac{-4}{5} \times \frac{11}{12}\right)$$

$$\frac{-4}{5} \times \frac{3+22}{24} = \frac{-1}{10} + \frac{-11}{15}$$

$$\frac{-4}{5} \times \frac{25}{24} = \frac{-3-22}{30}$$

SOLVED REVIEW EXERCISE 2

1. Answer the following questions.

(i) Define a rational number.

Answer: A number that can be expressed in the form of $\frac{p}{q}$ is called a rational number where p and q are integers and $q \neq 0$.

(ii) Write the additive inverse of the rational numbers "a".

Answer: $-a$ is the additive inverse of a , as $a + (-a) = 0$

(iii) What is the reciprocal of the rational number $\frac{p}{q}$, $q \neq 0$?

Answer: $\frac{q}{p}$ if $p \neq 0$

(iv) Write the sum of two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, $q, s \neq 0$.

Answer: $\frac{ps + rq}{qs}$

(v) What is the rule to find the product of two rational numbers?

Answer: Multiply the numerator of one rational number by the numerator of the other number. Similarly, multiply the denominators of both rational numbers.

(vi) What are the inverse operations of addition and multiplication?

Answer:

The inverse operation of addition is subtraction.

The inverse operation of multiplication is division.

2. Fill in the blank.

(i) The _____ consists of fractions as well as integers.

(ii) The rational numbers $\frac{p}{q}$ and $\frac{-p}{q}$ are called _____ inverse of each other.

(iii) A number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called the _____ number.

(iv) 0 is called additive identity whereas 1 is called _____ identity.

(v) The rational number $\frac{0}{p}$, $p \neq 0$ has no _____ $\forall p \in \mathbb{Q}$.

(vi) The _____ inverse of a rational number is its reciprocal.

Answers:

(i)	rational numbers	(ii)	additive	(iii)	rational
(iv)	Multiplicative	(v)	reciprocal	(vi)	multiplicative

$$\frac{-5}{6} = \frac{-25}{30}$$

$$\frac{-5}{6} = \frac{-5}{6}$$

$$(iii) \quad \frac{4}{9} \times \left(\frac{2}{3} \times \frac{5}{7} \right) = \left(\frac{4}{9} \times \frac{2}{3} \right) \times \frac{5}{7}$$

$$\frac{4}{9} \times \frac{10}{21} = \frac{8}{27} \times \frac{5}{7}$$

$$\frac{40}{189} = \frac{40}{189}$$

$$(iv) \quad \left(\frac{-121}{169} \right) \times \left(\frac{13}{-11} \right) = \left(-\frac{13}{11} \right) \times \left(\frac{-121}{169} \right)$$

$$\frac{11}{13} = \frac{11}{13}$$

$$(v) \quad \frac{-1}{4} + \left(\frac{1}{6} + \frac{3}{5} \right) = \left(\frac{-1}{4} + \frac{1}{6} \right) + \frac{3}{5}$$

$$\frac{-1}{4} + \frac{5+18}{30} = \frac{-3+2}{12} + \frac{3}{5}$$

$$\frac{-1}{4} + \frac{23}{30} = \frac{-1}{12} + \frac{3}{5}$$

$$\frac{-15+46}{60} = \frac{-5+36}{60}$$

$$\frac{31}{60} = \frac{31}{60}$$

$$(vi) \quad \frac{5}{12} \times \left(\frac{-2}{7} - 2 \right) = \left(\frac{5}{12} \times \frac{-2}{7} \right) - \left(\frac{5}{12} \times 2 \right)$$

$$\frac{5}{12} \times \frac{-2-14}{7} = \frac{-5}{42} - \frac{5}{6}$$

$$\frac{5}{12} \times \frac{-16}{7} = \frac{-5-35}{42}$$

$$\frac{-20}{21} = \frac{-40}{42}$$

$$\frac{-20}{21} = -\frac{20}{21}$$