CHAPTER-10
FUNDAMENTALS OF GEOMETRY

Students Learning Outcomes

After studying this chapter, students will be able to:

- Define adjacent, complementary and supplementary angles.
- Define vertically opposite angles,
- Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
- Calculate unknown angle of a triangle.
- Identify congruent and similar figures.
- Recognize the symbol of congruence.
- Apply the properties for *two* figures to be congruent or similar.
- Apply following properties for congruence between two triangles.
  - SSS ≅ SSS
  - SAS ≅ SAS
  - ASA ≅ ASA
  - RHS ≅ RHS
- Describe a circle and its centre, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
- Draw a semicircle and demonstrate the property; the angle in a semicircle is a right angle.
- Draw a segment of a circle and demonstrate the property; the angles in the same segment of a circle are equal.

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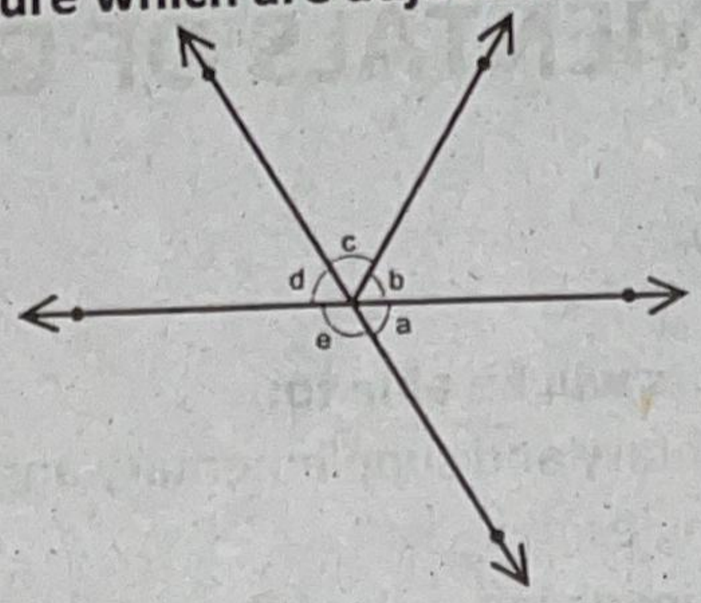
SOLVED EXERCISE 10.1

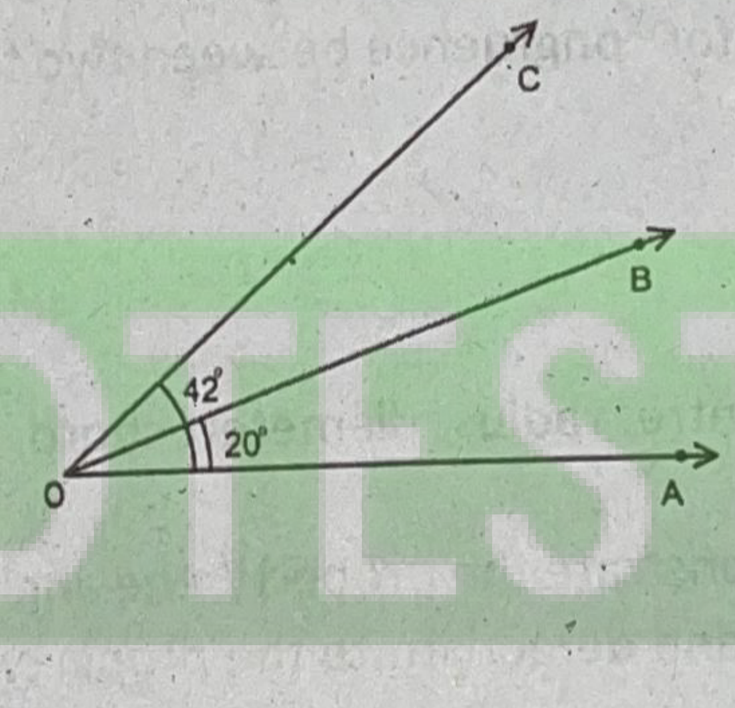
1. Name all the angles in the figure which are adjacent.



Solution:

∠a, ∠b; ∠b, ∠c; ∠c, ∠d;
∠d, ∠e; ∠e, ∠a

2. In the following figure *∠AOB* and *∠BOC* are adjacent angles, i.e. m∠AOB = 20° and *m∠AOC* = 42°. Find *m∠BOC*.



Solution:

m∠AOB = 20°, m∠AOC = 42°
m∠BOC = ?
We know that
m∠AOC = m∠AOB + m∠BOC
42° = 20° + m∠BOC
42° – 20° = m∠BOC
∠BOC= 22°

3. Identify the pairs of complementary and supplementary angles.

(i) 50°, 40°
50°+ 40° = 90° 50°and 40° are complementary angles.

(ii) 120°, 60°
120°+ 60° = 180° 120°and 60° are supplementary angles.

(iii) 70°, 70°
70°+ 70° = 140° 70° and 70° are supplementary neither nor complementary angles.

(iv) 130°,50°

$130° + 50° = 180°$ $130°$ and $50°$ are supplementary angles.

(v) $70°, 20°$

$70° + 20° = 90°$ $70°$ and $20°$ are complementary angles.

(vi) $50°, 100°$

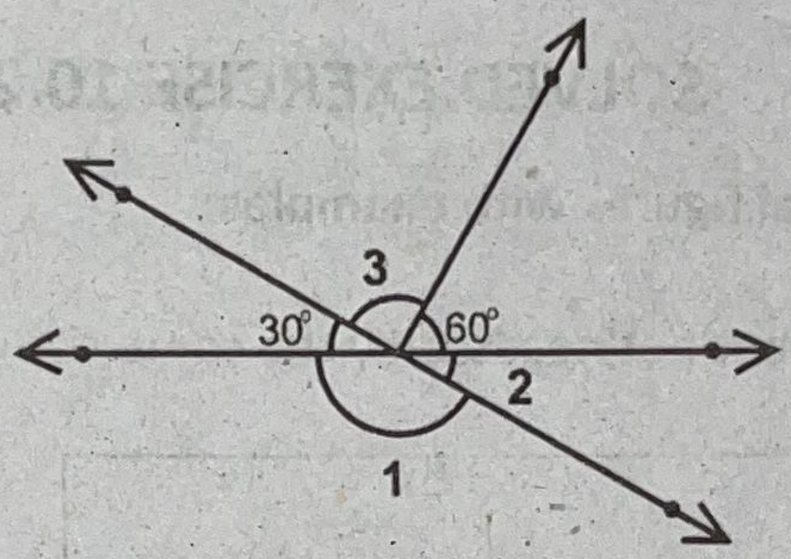
$50° + 100° = 150°$ $50°$ and $100°$ are supplementary neither nor complementary angles.

4. In the given figure, find all the remaining angles.

$m\angle 1 =$ _____

$m\angle 2 =$ _____

$m\angle 3 =$ _____



Solution:

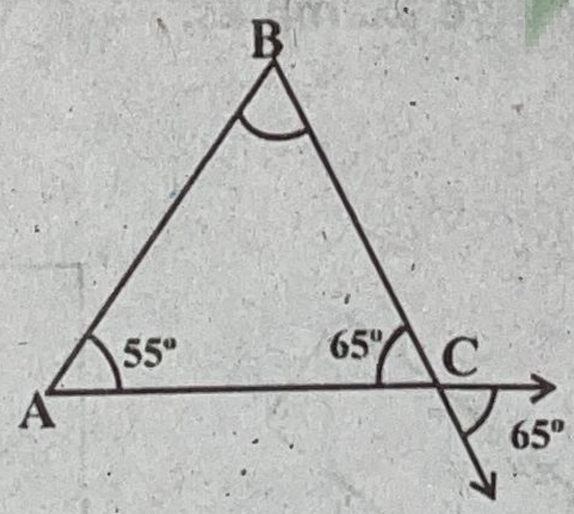
$m\angle 1 = 180° - 30° = 150°$

$m\angle 2 = 90° - 60° = 30°$

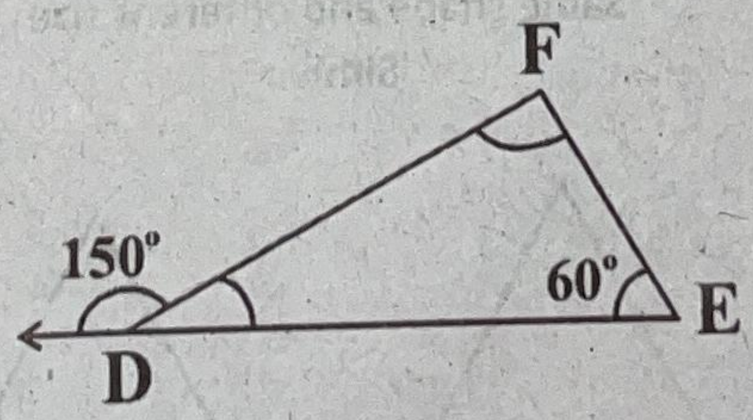
$m\angle 3 = 90°$

5. Find the unknown angles of the given triangles.

(i)



(ii)



Solution:

(i) $m\angle A = 55°, m\angle °C = 65°, m\angle B = ?$

We know that

$$m\angle A + m\angle B + m\angle C = 180^\circ$$
$$55^\circ + m\angle B + 65^\circ = 180^\circ$$
$$m\angle B = 180^\circ - 65^\circ - 55^\circ = 180^\circ - 120^\circ = 60^\circ$$

(ii) $m\angle E = 60^\circ$, $m\angle F = ?$,

$m\angle D = 180^\circ - 150^\circ = 30^\circ$

$$m\angle E + m\angle D + m\angle F = 180^\circ$$
$$60^\circ + 30^\circ + m\angle F = 180$$
$$m\angle F = 180^\circ - 90^\circ = 90^\circ$$

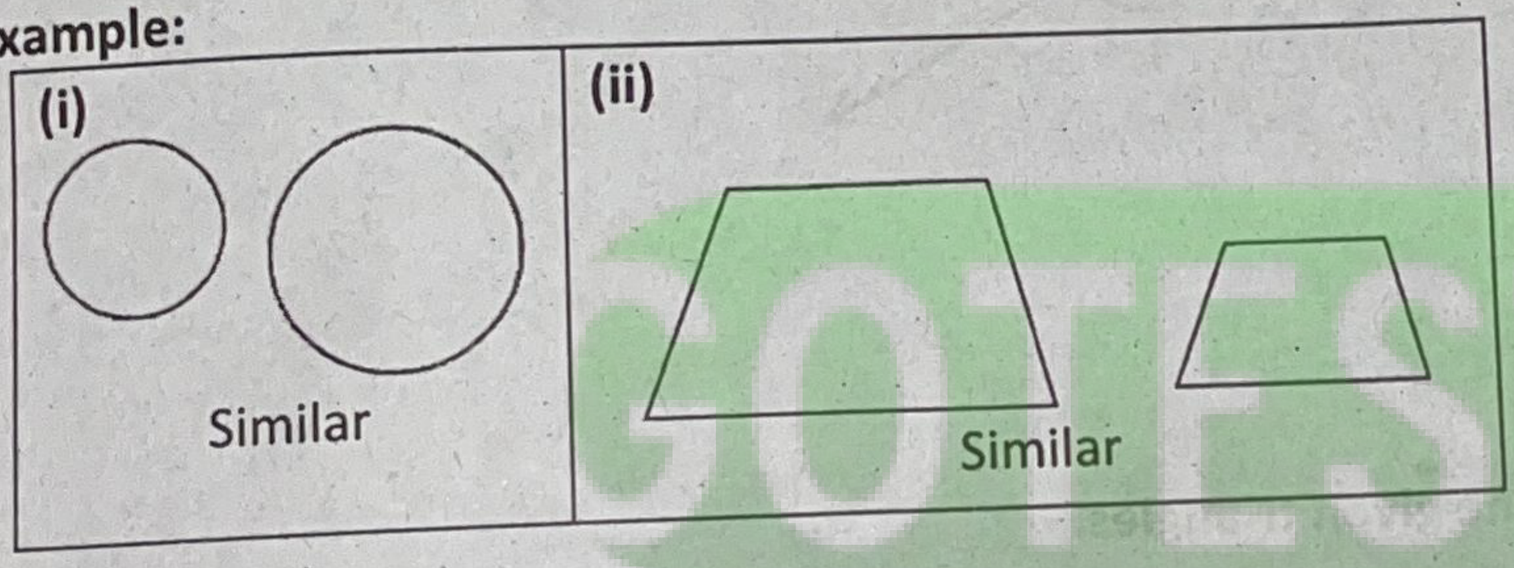
SOLVED EXERCISE 10.2

1. Define similar geometrical figures with examples.

Solution:

The figures with the same shape but not necessarily the same size are called similar figure.

Example:

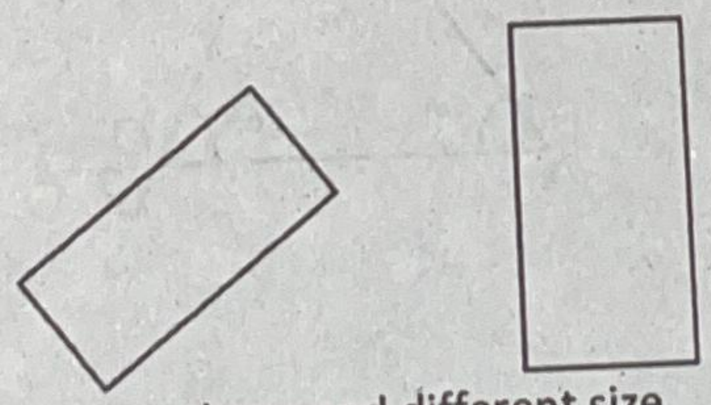


2. Are similar figures congruent? Give examples.

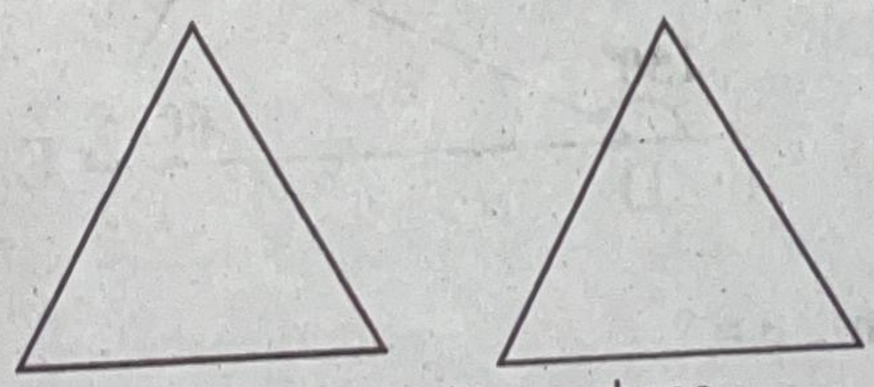
Solution: No

Example:

(i)



(ii)



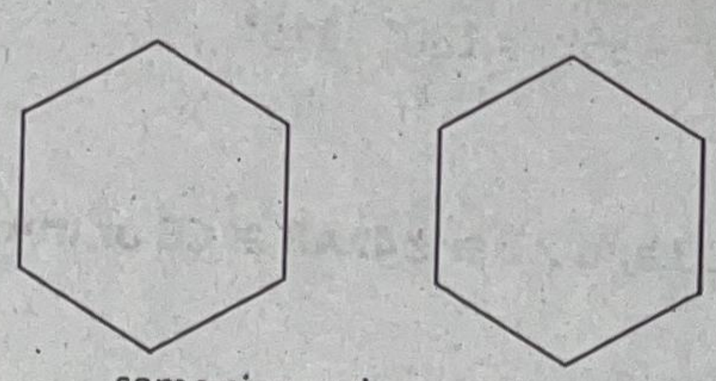
3. Are congruent figures similar? Prove this with examples.

Solution:

Yes

Example:

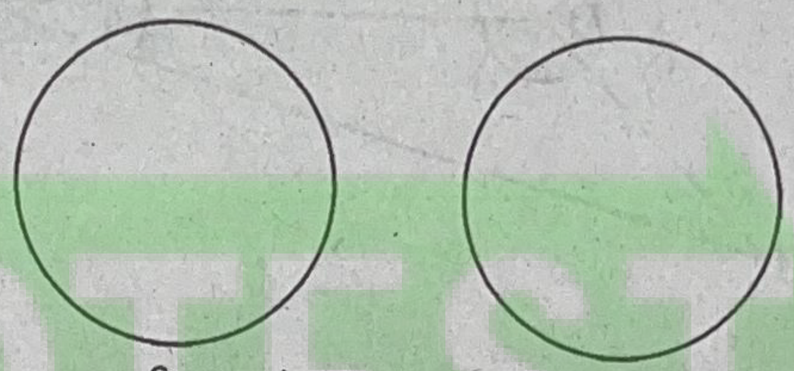
(i)



same size and same shape

Figures are congruent and Similar

(ii)

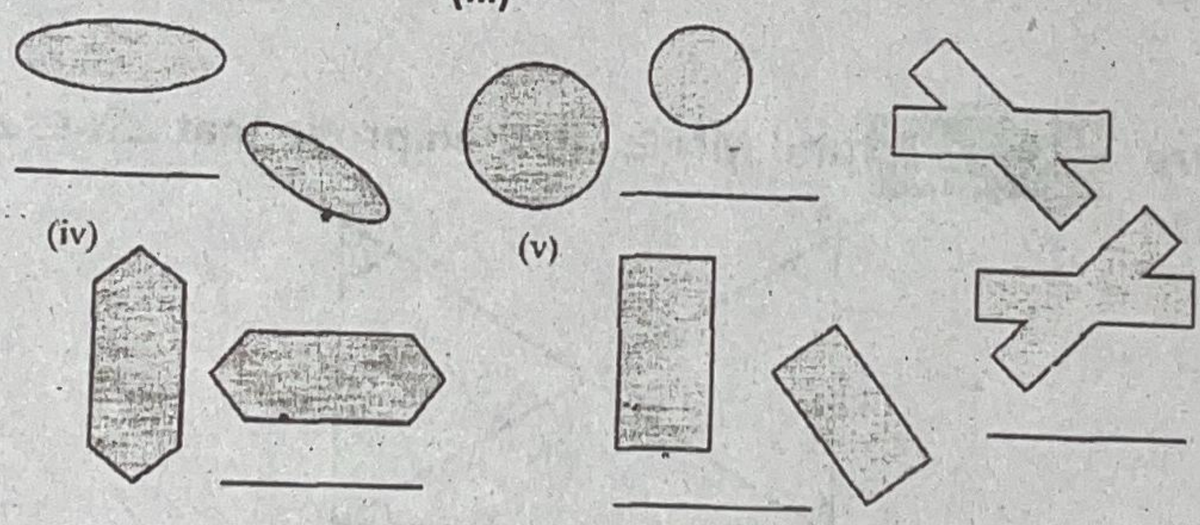


Same size and same shape

Figures are congruent and Similar

4. Identify congruent and similar pairs of figures.

(i) **(ii)** **(iii)**



Solution:

(i) Similar **(ii)** Similar

(iii) Congruent **(iv)** Congruent

(v) Similar

SOLVED EXERCISE 10.3

1. If the measures of two angles of a triangle are 35° and 80°, then find the measuring of its third angle.

Solution:

Measure of first angle = 35°, Measure of second angle = 80°

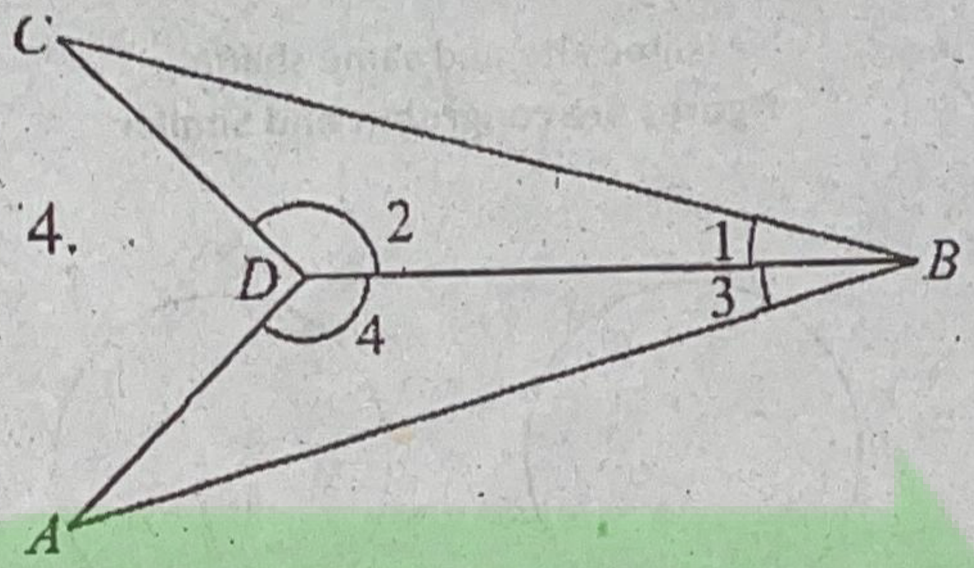
35° + 80° + Measure of third angle = 180°

Measure of third angle = 180° – 80° – 35° = 180° - 115°

Measure of third angle = 65°

2. In the given figure, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. AB ≅ CB or mAB = mCB Then prove that, ΔABD ≅ ΔBDC

4.



Solution:

Given that

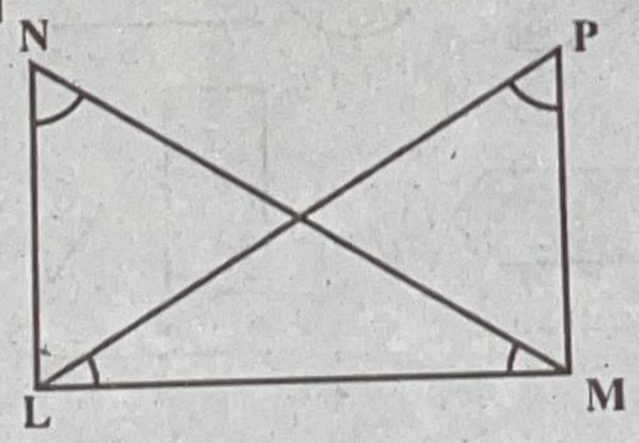
$\angle 1 \cong \angle 3$

$\angle 2 \cong \angle 4$

$\overline{BD} \cong \overline{BD}$ or $m\overline{BD} = m\overline{BD}$

Thus ΔABD ≅ ΔCBD

3. In the given figure, $\overline{LN} \cong \overline{MP}$ and MN ≅ LP, then prove that ∠N ≅ ∠P and ∠LMN ≅ ∠MLP



Solution:

ΔLMN and ΔLMP are two triangles.

Given that

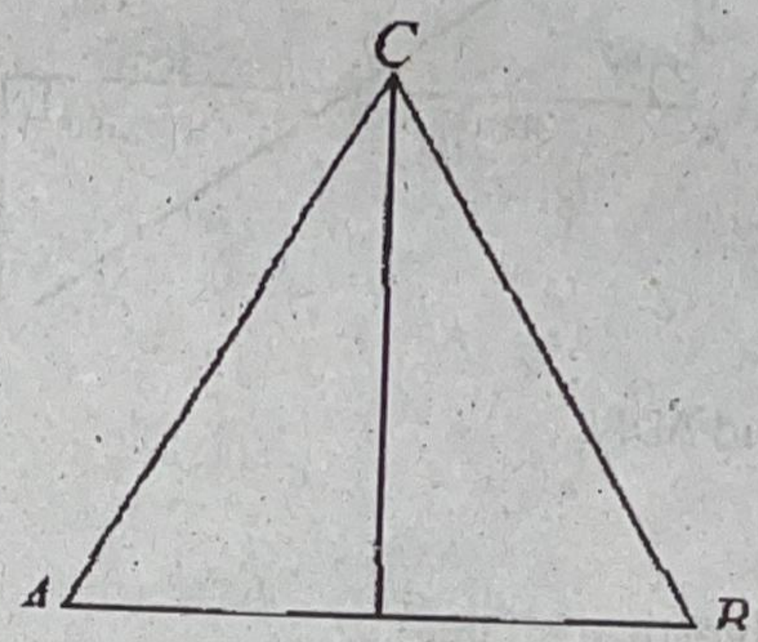
$\overline{LN} \cong \overline{MP}$

$\overline{LP} \cong \overline{MN}$

$\overline{LM} \cong \overline{LM}$ (common) $\Rightarrow \Delta LMN \cong \Delta LMP$

Thus ∠LMN ≅ ∠MLP and ∠N ≅ ∠P

4. In the given triangle ΔABC, $\overline{CD} \perp \overline{AB}$ and $\overline{CA} \cong \overline{CB}$ then prove that $\overline{AD} \cong \overline{BD}$ and ∠ACD ≅ ∠BCD



Solution:

Given that ΔABC has two triangles ΔADC and ΔBDC.

$\overline{CD} \perp \overline{AB} \Rightarrow m\angle ADC = m\angle BDC = 90°$

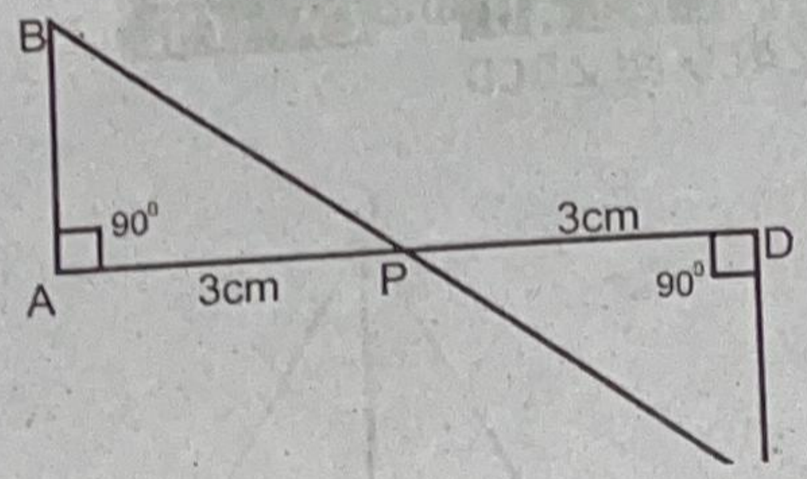
$\overline{CA} \cong \overline{CB}$ also $\overline{DC} \cong \overline{DC}$ (Common)

$\Rightarrow$ ΔADC ≅ ΔBDC

Thus $\overline{AD} \cong \overline{BD}$

and ∠ACD ≅ ∠BCD

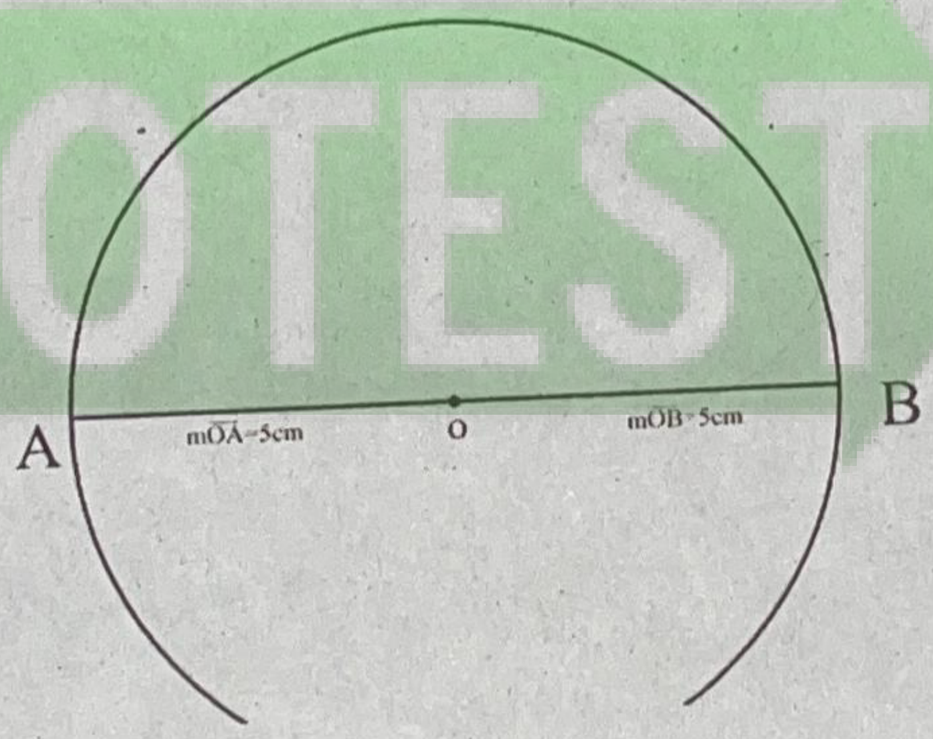
6. Look at the figure to show that $\Delta ABP = \Delta DCP$



Solution:
According to the figure in ΔAPB and ΔDPC
$m\overline{AP} \cong mDP$
$m\angle PAB \cong m\angle PDC$
also $m\angle APB \cong m\angle DPC$ (vertical angles)
$\Rightarrow \Delta APB \cong \Delta DPC$

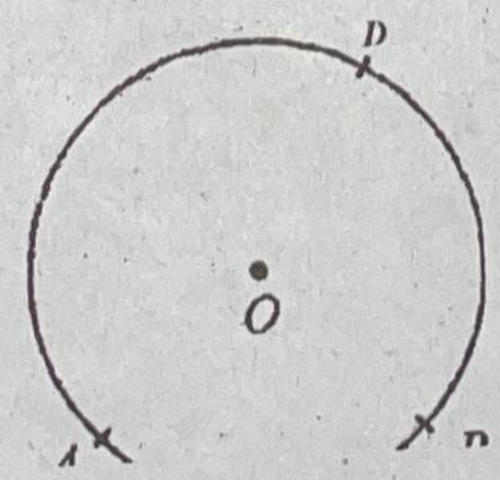
SOLVED EXERCISE 10.4

1. Draw a circle with radius $\overline{OA}$ = 5cm, and find its diameter.



Ans: Def of diameter:
A line segment that passes through the center of a circle and meets two points on its circumference is called the diameter of the circle. In the given figure $\overline{AB}$ is the diameter of the circle.
Measure of the diameter, twice is the radius, that is, it is of 10 cm.

2. In the given figure locate major and minor arcs.



Ans: Minor Arc:

An arc having its length smaller than half the circumference of a circle is called minor arc. It is named by using two end points of the arc. In figure AB and BD are minor ares.

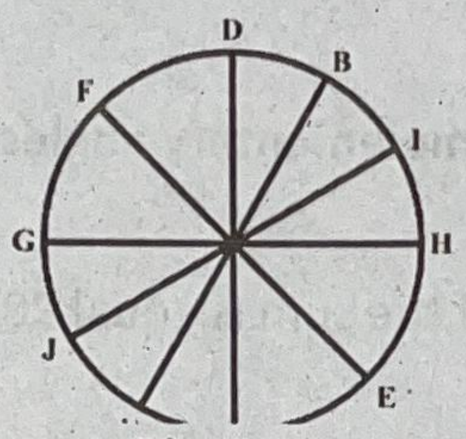
Major Arc:

An arc having its length greater than half the circumference of a circle is called major arc. It is named by three points. In figure ABD is a major arc.

3. How many diameters of a circle can be drawn. Draw a circle and trace at least '5' different diameters.

Solution:

Infinitely many diameters can be drawn. $\overline{AB}, \overline{CD}, \overline{EF}, \overline{HC}$ and $\overline{IJ}$ are five diameters shown in the figure.



Diameters

4. Draw a semicircle of a radius of 2cm.

$m\overline{0A} = 2cm = m\overline{0B}$

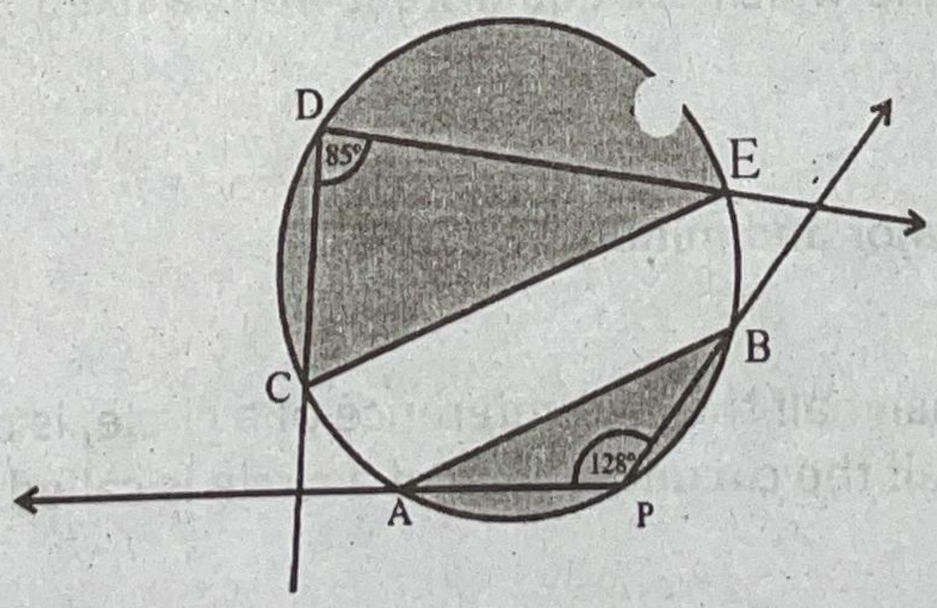
5. Draw a circle and cut it into two segments. Construct two inscribed angles in each of the segment and measure them.

Solution:

Two shaded segments are shown in the figure.

$m\angle APB = 128°$

$m\angle CDE = 85°$



SOLVED REVIEW EXERCISE 10

1. Answer the following questions.

(i) What is meant by the adjacent angles?

Solution:

Two angles are said to the adjacent if

(i) They have same vertex,

(ii) They have one common arm.

(iii) Other arms of two angles are on opposite sides of the common arm.

(ii) What is the difference between complementary angles and supplementary angles?

Solution:

Two angles are said to be complementary if the sum is equal 90°. Two angles are said to be supplementary if the sum is equal to 180°.

(iii) Define the vertically opposite angles.

Solution:

Angles are said to be vertically opposite if the angles arc formed by two intersecting lines and the angles arc non-adjacent.

(iv) What is the symbol of congruency?

Solution:

The symbol of congruence is $\cong$.

(v) What is a circle?

Solution:

A circle is a set of points in a plane which arc equidistant from a fixed point called the center of the circle.

(vi) Differentiate between .major and minor arcs.

Solution:

An arc having its length smaller than half the circumference of a circle, is called a minor arc. An arc having its length is greater than half the circumference of a circle is called a major arc.

2. Fill in the blanks.

(i) From the adjacent angles we mean, angles next to each other.

(ii) If the sum of two angles is 90° then the angles are called complementary angles.

(iii) The non-adjacent angles which are formed by two intersecting lines are called vertical opposite angles.

(iv) Two figures are congruent if they are the same in shape and size.

(v) A circle is a set of points which are equidistant from a fixed point, called its centre.

(vi) Two. triangles are congruent, if three sides of one triangle are congruent to the three sides of another triangle.

(vii) The figures with the same shape but not necessarily the same size are called similar figures.

(viii) Vertically opposite angles are always equal in measure.

3. Tick (✓) the correct option.

(i) The symbol for congruence of figures is:

(a) ° (b) ^ (c) ✓ ≅ (d) ∩

(ii) A line segment joining two points on a circle is called;

(a) ✓ Chord (b) arc (c) radius (d) diameter

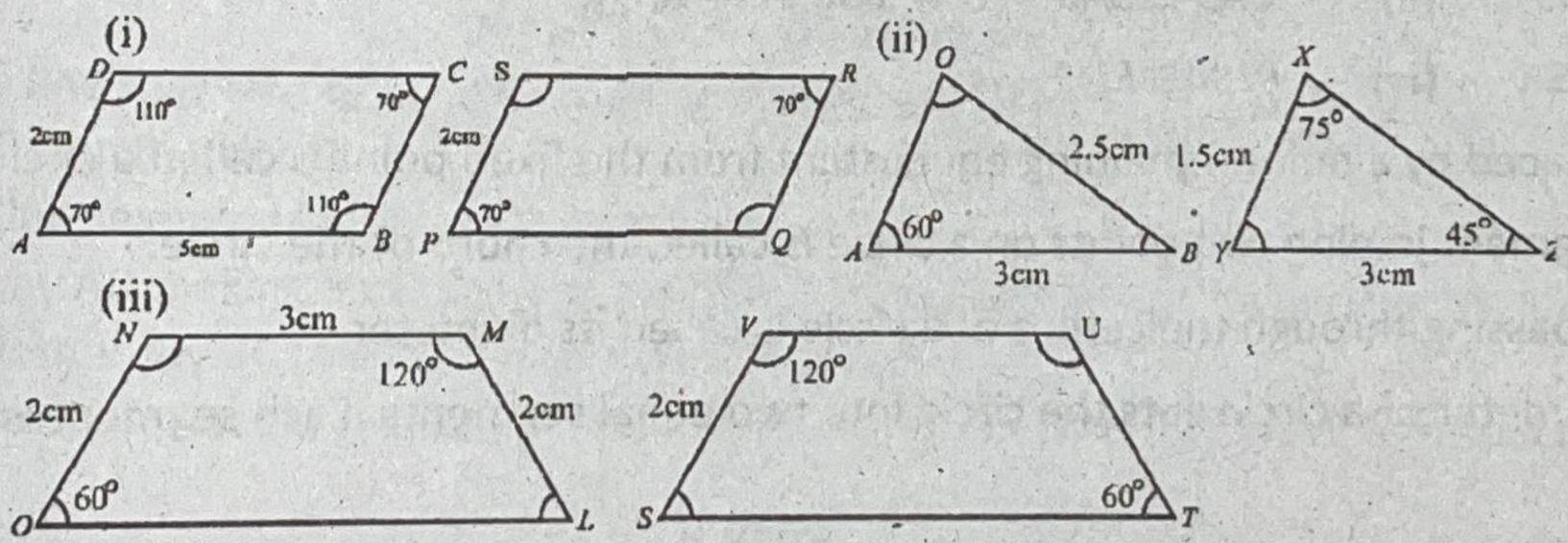
(iii) In a pair of complementary angles, if the measure of one angle is 45°, then the measure of the other angle will be:

(a) 30° (b) ✓ 45° (c) 60° (d) 90°

(iv) In a pair of supplementary angles, if the measure of one angle is 60°, then the measure of the other angle will be:

(a) 45° (b) 90° (c) ✓ 120° (d) 180°

4. Find unknown measures of the sides and angles for these congruent shapes.



Solution:

(i) $m\overline{DC} = 5cm$, $m\overline{BC} = 2cm$, $m\overline{PQ} = 5cm$, $m\overline{SR} = 5cm$,

$m\overline{RQ} = 2cm$, $m\angle S = 110°$, $m\angle Q = 110°$.

(ii) $m\overline{OA} = 1.5cm$, $m\angle B = 45°$, $m\angle O = 75°$, $m\overline{XZ} = 2.5cm$, $m\angle Y = 60°$.

(iii) $m\overline{OL} = 5cm$, $m\angle N = 120°$, $m\angle L = 60°$, $m\overline{UT} = 2cm$, $m\overline{UV} = 3cm$, $m\angle S = 60°$, $m\angle U = 120°$.

5. If a and b are complementary angles then find the value of b if measure of a is 40°.

Solution: $a = 40, b = ?$

$a + b = 90 \Rightarrow 40 + b = 90° \Rightarrow b = 90° - 40° \Rightarrow b=50$

6. If x and y are two supplementary angles where $m\angle x = 60°$, then find the measure of y.

Solution:

$m\angle x + m\angle Y = 180° \Rightarrow m\angle Y + 60° = 180°$

$m\angle Y = 180 - 60°$

$\angle Y = 120°$

SUMMARY

- Two angles with a common vertex and one common arm, are called adjacent angles.
- If the sum of two adjacent angles is 90°, then the angles are called complementary angles.
- If the sum of two adjacent angles is 180°, then the angles will be supplementary.
- If two lines intersect each other, the non adjacent angles, so formed, are called vertical angles.
- Two geometrical figures are similar if they are the same in shape.
- Figures are congruent if they are the same in shape and size.
- Two triangles will be congruent if any of the following property holds:

(i) $SSS \cong SSS$ (ii) $SAS \cong SAS$ S ⇒ side, A ⇒ Angle

(iii) $ASA \cong ASA$ (iv) $RHS \cong RHS$

- A path traced by a point remaining equidistant from the fixed point is called a circle.
- A line segment joining two points on a circle is called the chord of the circle.
- A chord passing through the centre of a circle is called its diameter.
- Every diameter of a circle cuts the circle into two equal segments. Each segment is called a semi-circle.
- An angle inscribed in a semicircle is a right angle.
- The angles in the same segment of a circle are equal in measure.