CHAPTER-4
EXPONENTS

Students Learning Outcomes

After studying this chapter, students will be able to:

(i) Identify base, exponent and value.

(ii) Use rational numbers to deduce laws of exponents.

➢ Product Law:

- When bases are same but exponents are different:

  ➢ $a^m \times a^n = a^{m+n}$

- When bases are different but exponents are same:

  ➢ $a^n \times b^n = (ab)^n$

➢ Quotient Law:

i. When bases are same but exponents are different:

➢ $a^m \div a^n = a^{m-n}$

ii. When bases are different but exponents are same:

➢ $a^n \div b^n = \left(\frac{a}{b}\right)^n$

➢ Power law: $(a^m)^n = a^{mn}$

➢ For zero exponent: $a^o = 1$

➢ For exponent as negative integer: $a^{-m} = \frac{1}{a^m}$

(iii) Demonstrate the concept of power of integer that is $(-a)^n$ when n is even or odd integer.

(iv) Apply laws of exponents to evaluate expressions.

SOLVED EXERCISE 4.1

1. Identify the exponent and base in each of the following.

Solution:

		Base	Exponent
(i)	$(-1)^9$	−1	9
(ii)	2^{100}	2	100
(iii)	$(-19)^{22}$	−19	22
(iv)	3^{-5}	3	−5
(v)	$(ab)^n$	ab	n
(vi)	$\left(-\frac{6}{11}\right)^8$	(-6 / 11)	8
(vii)	a^{-mn}	a	−mn
(viii)	$\left(\frac{2}{9}\right)^7$	2/7	7
(ix)	$\left(\frac{p}{q}\right)^4$	p/q	4
(x)	$\left(\frac{-1}{x}\right)^6$	− 1/x	6
(xi)	$\left(\frac{x}{y}\right)^m$	x/y	m
(xii)	$\left(\frac{11}{13}\right)^{-6}$	11/13	−6

2. Express each of the following in exponential form.

Solution:

(i) $5\times5\times5\times5=5^4$

(ii) $\frac{-3}{7}\times\frac{-3}{7}\times\frac{-3}{7}\times\frac{-3}{7}=\left(\frac{-3}{7}\right)^4$

(iii) $p\times p\times p\times p\times p=p^5$

(iv) $\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}=\left(\frac{1}{10}\right)^3$

(v) $xy\times xy\times xy=(xy)^3$

(vi) $31\times31\times31\times31\times31=(31)^5$

(vii) $(-a)\times(-a)\times(-a)\times(-a)\times(-a)\times(-a)\times(-a)=(-a)^7$

3. Prove that

Solution:

(i) $(5)^3 = 5\times5\times5 = 125$

(ii) $(-1)^{11} = (-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1) = -1$

(iii) $(-3)^5 = -3\times-3\times-3\times-3\times-3 = -243$

(iv) $\left(\frac{3}{7}\right)^2 = \frac{3}{7}\times\frac{3}{7} = \frac{9}{49}$

(v) $\left(-\frac{1}{8}\right)^3 = \left(-\frac{1}{8}\right)\times\left(-\frac{1}{8}\right)\times\left(-\frac{1}{8}\right) = -\frac{1}{512}$

(vi) $\left(\frac{-2}{3}\right)^6 = \frac{-2}{3}\times\frac{-2}{3}\times\frac{-2}{3}\times\frac{-2}{3}\times\frac{-2}{3}\times\frac{-2}{3} = \frac{64}{729}$

(vii) $\left(\frac{1}{10}\right)^4 = \frac{1}{10\times10\times10\times10} = \frac{1}{10000}$

(viii) $\left(-\frac{4}{3}\right)^3 = \frac{-4}{3}\times\frac{-4}{3}\times\frac{-4}{3} = \frac{-64}{27}$

(ix) $\left(\frac{2}{5}\right)^4 = \frac{2\times2\times2\times2}{5\times5\times5\times5} = \frac{16}{625}$

4. Express each rational number using an exponent.

Solution:

(i) $121 = 11\times11 = 11^2$

(ii) $81 = 3\times3\times3\times3 = 3^4$

(iii) $-625 = -(5\times5\times5\times5) = -(5)^4$

(iv) $1/1000 = \left(\frac{1}{10}\right)^3$

(v) $= = \frac{2^3}{7^3} = \left(\frac{2}{7}\right)^3$

(vi) $-\frac{1}{32} = - = \frac{-1}{2^5} = \left(-\frac{1}{2}\right)^5$

SOLVED EXERCISE 4.2

1. Simplify by using the laws of exponent into the exponential form.

Solution:

(i) $(-4)^5 \times (-4)^6 = (-4)^{5+6} = (-4)^{11}$

(ii) $m^3 \times m^4 = m^7$

(iii) $\left(\frac{2}{7}\right)^3 \times \left(\frac{2}{7}\right)^2 = \left(\frac{2}{7}\right)^{3\times2} = \left(\frac{2}{7}\right)^5$

(iv) $\left(\frac{1}{10}\right)^4 \times \left(\frac{1}{10}\right)^5 = \left(\frac{1}{10}\right)^{4+5} = \left(\frac{1}{10}\right)^9$

(v) $p^{10} \times q^{10} = p^{10}q^{10} = (pq)^{20}$

(vi) $\left(\frac{2}{5}\right)^3 \times \left(\frac{5}{7}\right)^3 = \frac{2^3}{5^3} \times \frac{5^3}{7^3} = \frac{2^3}{7^3} = \left(\frac{2}{7}\right)^3$

(vii) $\left(\frac{-1}{2}\right)^6 \times \left(\frac{-1}{2}\right)^5 = \left(\frac{-1}{2}\right)^{11}$

(viii) $(-3)^7 \times (-5)^7 = (-3 \times -5)^7 = (15)^7$

(ix) $\left(\frac{2}{3}\right)^{10} \times \left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^{17}$

(x) $\left(\frac{-10}{11}\right)^7 \times \left(\frac{-10}{11}\right)^6 = \left(\frac{-10}{11}\right)^{13}$

(xi) $\left(\frac{11}{7}\right)^8 \times \left(\frac{21}{22}\right)^8 = \left(\frac{11}{7} \times \frac{21}{22}\right)^8 = \left(\frac{3}{2}\right)^8$

(xii) $\left(\frac{-x}{y}\right) \times \left(\frac{-x}{y}\right)^{11} = \left(\frac{-x}{y}\right)^{12}$

2. . Verify the following by using the laws of exponent.

Solution:

(i) $(3\times5)^4 = 3^4 \times 5^4$

$(15)^4 = (3\times5)^4$

$(15)^4 = (15)^4$

(ii) $(7\times9)^8 = 7^8 \times 9^8$

$(7\times9)^8 = (7\times9)^8$

$(63)^8 = (63)^8$

(iii) $(2)^6 \times (2)^3 = 2^9$

$(2)^{6+3} = 2^9$

$2^9 = 2^9$

(iv) $(x\times y)^m = x^m y^m$

$x^m \times y^m = x^m y^m$

$x^m y^m = x^m y^m$

(v) $(8)^5 \times (8)^7 = (8)^{12}$

$(8)^{5+7} = (8)^{12}$

$(8)^{12} = (8)^{12}$

(vi) $(p)^r \times (p)^s = p^{r+s}$

$(p)^{r+s} = p^{r+s}$

$p^{r+s} = p^{r+s}$

SOLVED EXERCISE 4.3

1. Simplify

Solution:

(i) $2^7 \div 2^2 = \frac{2^7}{2^2} = 2^7 \times 2^{-2} = 2^5$

(ii) $(-9)^{11} \div (-9)^8 = \frac{(-9)^{11}}{(-9)^8} = (-9)^{11}(-9)^{-8} = (-9)^3$

(iii) $(3)^4 \div (5)^4 = \frac{3^4}{5^4} = \left(\frac{3}{5}\right)^4$

(iv) $m^3 \div n^3 = \frac{m^3}{n^3} = \left(\frac{m}{n}\right)^3$

(v) $a^7 \div a^2 = \frac{a^7}{a^2} = a^7 . a^{-2} = a^5$

(vi) $(b)^p \div (b)^q = \frac{(b)^p}{(b)^q} = b^p.b^{-q} = b^{p-q}$

(vii) $\left(\frac{3}{4}\right)^7 \div \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^7 \times \left(\frac{3}{4}\right)^{-2} = \left(\frac{3}{4}\right)^5$

(viii) $\left(\frac{1}{6}\right)^{15} \div \left(\frac{1}{6}\right)^{11} = \left(\frac{1}{6}\right)^{15} \times \left(\frac{1}{6}\right)^{-11} = \left(\frac{1}{6}\right)^4$

(ix) $(2)^5 \div (3)^5 = \frac{(2)^5}{(3)^5} = \left(\frac{2}{3}\right)^5$

(x) $\left(\frac{-3}{10}\right)^{17} \div \left(\frac{-3}{10}\right)^8 = \left(\frac{-3}{10}\right)^{17} \times \left(\frac{-3}{10}\right)^{-8} = \left(\frac{-3}{10}\right)^9$

(xi) $(x)^9 \div (y)^9 = \frac{x^9}{y^9} = \left(\frac{x}{y}\right)^9$

(xii) $\left(\frac{p}{q}\right)^{23} \div \frac{p}{q} = \left(\frac{p}{q}\right)^{23} \times \left(\frac{p}{q}\right)^{-1} = \left(\frac{p}{q}\right)^{22}$

2. Prove that

Solution:

(i) $2^4 \div 7^4 = \left(\frac{2}{7}\right)^4$

L.H.S $= 2^4 \div 7^4 = = \left(\frac{2}{7}\right)^4 =$ R.H.S

(ii) $(-4)^3 \div (5)^3 = \left(-\frac{4}{5}\right)^3$

L.H.S $= (-4)^3 \div (5)^3 = \frac{(-4)^3}{(5)^3} = \left(-\frac{4}{5}\right)^3 =$ R.H.S

(iii) $3^8 \div 3 = 3^7$

L.H.S $= 3^8 \div 3 = \frac{3^8}{3} = 3^8 \times 3^{-1} = 3^7$ = R.H.S

(iv) $a^6 \div b^6 = \left(\frac{a}{b}\right)^6$

L.H.S $= a^6 \div b^6 = \frac{a^6}{b^6} = \left(\frac{a}{b}\right)^6$ = R.H.S

(v) $\left(\frac{-21}{22}\right)^7 \div \left(\frac{-21}{22}\right)^3 = \left(\frac{-21}{22}\right)^4$

L.H.S $= \left(\frac{-21}{22}\right)^7 \div \left(\frac{-21}{22}\right)^3 = \left(\frac{-21}{22}\right)^7 \times \left(\frac{-21}{22}\right)^{-3} = \left(\frac{-21}{22}\right)^4$ = R.H.S

(vi) $\left(\frac{-9}{13}\right)^5 \div \left(\frac{-9}{13}\right)^4 = \left(\frac{-9}{13}\right)$

L.H.S $= \left(\frac{-9}{13}\right)^5 \div \left(\frac{-9}{13}\right)^4 = \left(\frac{-9}{13}\right)^5 \times \left(\frac{-9}{13}\right)^{-4} = \left(\frac{-9}{13}\right)$ = R.H.S

SOLVED EXERCISE 4.4

1. Express the following as single exponents.

Solution:

(i) $(2^3)^5 = 2^{15}$
$= 2^{3\times5} = 2^{15}$

(ii) $(10^2)^2 = 10^4$

(iii) $(-3^4)^5 = (-3)^{20}$

(iv) $(p^2)^3 = p^6$

(v) $(-m^7)^4 = (-m)^{28}$

(vi) $(x^a)^b = (x)^{ab}$

(vii) $\left[\left(-\frac{1}{3}\right)^3\right]^3 = \left(-\frac{1}{3}\right)^9$

(viii) $\left[\left(\frac{2}{9}\right)^3\right]^6 = \left(\frac{2}{9}\right)^{18}$

(ix) $\left[\left(\frac{p}{q}\right)^m\right]^n = \left(\frac{p}{q}\right)^{mn}$

2. Change the following negative exponent into positive exponents.

Solution:

(i)
$$(12)^{-3}=\frac{1}{(12)^{3}}=\left(\frac{1}{12}\right)^{3}$$

(ii)
$$(-a)^{-2}=\frac{1}{(-a)^{2}}=\left(\frac{1}{-a}\right)^{2}$$

(iii)
$$(100)^{-5}=\frac{1}{(100)^{+5}}=\left(\frac{1}{100}\right)^{5}$$

(iv)
$$\left(\frac{2}{3}\right)^{-4}=\frac{2^{-4}}{3^{-4}}=\frac{3^{4}}{2^{4}}=\left(\frac{3}{2}\right)^{4}$$

(v)
$$\left(\frac{-1}{10}\right)^{-9}=\frac{(-1)^{-9}}{(10)^{-9}}=\frac{(10)^{9}}{(-1)^{9}}=\left(\frac{10}{-1}\right)^{9}=(-10)^{9}$$

(vi)
$$\left(\frac{x}{y}\right)^{-b}=\frac{x^{-b}}{y^{-b}}=\frac{y^{b}}{x^{b}}=\left(\frac{y}{x}\right)^{b}$$

3. Evaluate the following expressions.

Solution:

(i)
$$\left(1^{2}\right)^{3}\times\left(2^{3}\right)^{2}=(1)^{6}\times(2)^{6}=1\times 64=64$$

(ii)
$$\left[(-3)^{7}\right]^{0}\times\left[(-3)^{2}\right]^{2}=(-3)^{0}\times(-3)^{4}=1\times 81=81$$

(iii)
$$\left[\left(\frac{-3}{4}\right)^{0}\right]^{3}\times\left[\left(\frac{-3}{4}\right)^{2}\right]^{2}=\left(\frac{-3}{4}\right)^{0}\times\left(\frac{-3}{4}\right)^{4}=1\times\frac{81}{256}=\frac{81}{256}$$

(iv)
$$\left(\frac{2^{3}}{2^{6}\div 2^{3}}\right)=\frac{2^{3}}{\frac{2^{6}}{2^{3}}}=\frac{2^{3}\times 2^{3}}{2^{6}}=\frac{2^{6}}{2^{6}}=1$$

(v)
$$\frac{\left(\frac{1}{2}\right)^{-3}\times\left(\frac{1}{2}\right)^{-6}}{\left(\frac{1}{2}\right)^{-5}}=\frac{\left(\frac{1}{2}\right)^{-9}}{\left(\frac{1}{2}\right)^{-5}}=\left(\frac{1}{2}\right)^{-9}\times\left(\frac{1}{2}\right)^{5}=\left(\frac{1}{2}\right)^{-4}=\frac{1}{2^{-4}}=1\times 2^{4}=16$$

(vi) $$\frac{\left(\frac{-2}{9}\right)^5 \times \left(\frac{-2}{9}\right)^{-5}}{\left(\frac{3}{2}\right)^4 \times \left(\frac{3}{2}\right)^{-4}} = \frac{\left(\frac{-2}{9}\right)^0}{\left(\frac{3}{2}\right)^0} = \frac{1}{1} = 1$$

(vii) $$\frac{\left(\frac{1}{3}\right)^{-3}\left(\frac{1}{3}\right)^{-5}}{\left(\frac{1}{3}\right)^{-4}\left(\frac{1}{3}\right)^{-6}} = \frac{\left(\frac{1}{3}\right)^{-8}}{\left(\frac{1}{3}\right)^{-10}} = \left(\frac{1}{3}\right)^{-8} \times \left(\frac{1}{3}\right)^{10} = \left(\frac{1}{3}\right)^2 = \frac{1}{3^2} = \frac{1}{9}$$

(viii) $$\frac{\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^{-4} \div \left(\frac{2}{3}\right)^{-4}} = \frac{\left(\frac{2}{3}\right)^{-1}}{\left(\frac{2}{3}\right)^{-4} \times \left(\frac{2}{3}\right)^4} = \frac{\frac{3}{2}}{\left(\frac{2}{3}\right)^0} = \frac{\frac{3}{2}}{1} = \frac{3}{2}$$

(ix) $$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^0 \times \left(\frac{2}{3}\right)^{-3} = \left(\frac{2}{3}\right)^0 = 1$$

(x) $$\left(\frac{-1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-4} = \frac{(2)^2}{(-1)^2} + \frac{(3)^3}{(1)^3} + \frac{(4)^4}{(1)^4} = 4 + 27 + 256 = 287$$

SOLVED REVIEW EXERCISE 4

1. Answer the following questions.

(i) What is meant by the exponent of a number?

Answer:

The exponent of a number tells us, how many times a number (base) is multiplied with itself.

(ii) What is the product law with the same base?

Answer:

$a^m \times a^n = a^{m+n}$

(iii) Define the power law of exponent.

Answer:

The base remains the same with a new exponent equal to product of the two exponent e.g., $(a^m)^n = a^{mn}$

(iv) What is the reciprocal of ?

Answer:

2. Fill in the blank.

(i) $5\times5\times5\times5$ can be written in exponential form as _____.

(ii) $a^n \times b^n$ = _____.

(iii) $a^n \div b^n$ = _____

(iv) Any non-zero rational number with _____exponent equals to 1.

(v) $(-a)^n$ is positive, if 'n' is an_____ integer.

Answers:

(i)	5^4	(ii)	$(ab)^n$	(iii)	$\left(\frac{a}{b}\right)^n$	(iv)	zero	(v)	even

3. Tick (✓) the correct answer.

(i) '3^{rd} power of 5' can be written as:

(a) 3^5 (b) 5^3 (c) 3×5 (d) $5 \div 3$

(ii) $(3^0+2^0) \div 7^0$ = ?

(a) $(5/7)^\circ$ (b) $(30)^\circ$ (c) 2 (d) All of the above

(iii) The reciprocal of $\left(\frac{p}{q}\right)^{-m}$ is:

(a) $\left(\frac{p}{q}\right)^m$ (b) $\left(\frac{q}{p}\right)^m$ (c) $\left(\frac{q}{p}\right)^{-m}$ (d) $\left(\frac{1}{pq}\right)^m$

(iv) $(-a)^n$ is negative, if n is:

(a) prime (b) even (c) composite (d) odd

(v) $a^m \div a^n$ =?

(a) a^{m+n} (b) a^{mn} (c) a^{m-n} (d) $a^{m/n}$

Answers:

(i)	b	(ii)	d	(iii)	b	(iv)	d	(v)	c

4. Find the values of:

Solution:

(i) $(4)^{-3} = \frac{1}{4^3} = \frac{1}{64}$

(ii) $(-5)^4 = 625$

(iii) $(2)^{-9} = \frac{1}{(2)^9} = \frac{1}{512}$

(iv) $\left(\frac{-1}{3}\right)^5 = \frac{(-1)^5}{3^5} = \frac{-1}{243}$

(v) $\left(\frac{3}{10}\right)^3 = \frac{(3)^3}{(10)^3} = \frac{27}{1000}$

(vi) $-\left(\frac{11}{13}\right)^2 = -\frac{121}{169}$

5. Use the Laws of exponents to find the value of x.

Solution:

(i) $\left[(-7)^3\right]^6 = 7^x$

$(-7)^{18} = 7^x \Rightarrow (-1)^{18}(7)^{18} = 7^x \Rightarrow (1)(7)^{18} = 7^x$

$7^{18} = 7^x \Rightarrow x = 18$

(ii) $\left[\left(\frac{3}{4}\right)^2\right]^5 = \frac{3^x}{4^x}$

$\left(\frac{3}{4}\right)^{10} = \frac{3^x}{4^x} \Rightarrow \left(\frac{3}{4}\right)^{10} = \left(\frac{3}{4}\right)^x \Rightarrow x = 10$

(iii) $\left[\left(\frac{13}{8}\right)^4\right]^4 = \frac{13^x}{8^x}$

$\left(\frac{13}{8}\right)^{16} = \left(\frac{13}{8}\right)^x \Rightarrow x = 16$

(iv) $\left(\frac{5}{3}\right)^5 \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$

$\left(\frac{5}{3}\right)^{16} = \left(\frac{5}{3}\right)^{8x} \Rightarrow 16 = 8x \Rightarrow \boxed{x = 2}$

(v) $\left(\frac{2}{9}\right)^2 \div \left(\frac{2}{9}\right)^9 = \left(\frac{2}{9}\right)^{2x-1}$

$\left(\frac{2}{9}\right)^2 \times \left(\frac{2}{9}\right)^{-9} = \left(\frac{2}{9}\right)^{2x-1}$

$$\left(\frac{2}{9}\right)^{-7}=\left(\frac{2}{9}\right)^{2x-1}$$

$$-7=2x-1 \quad \Rightarrow \quad -2x=6 \Rightarrow \boxed{x=-3}$$

6. Simplify and write the answer in simple form:

Solution:

(i) $$\left[\left(\frac{-3}{4}\right)^{2}\times\left(\frac{-3}{4}\right)^{3}\right]\div\left[\left(\frac{-3}{4}\right)^{2}\right]^{2}$$

$$=\left(\frac{-3}{4}\right)^{5}\div\left(\frac{-3}{4}\right)^{4}=\left(\frac{-3}{4}\right)^{5}\times\left(\frac{-3}{4}\right)^{-4}=\left(\frac{-3}{4}\right)$$

(ii) $$\left(\frac{5}{19}\right)^{10}\times\left[\left(\frac{5}{19}\right)^{2}\right]^{3}\div\left[\left(\frac{5}{19}\right)^{4}\right]^{4}$$

$$=\left(\frac{5}{19}\right)^{10}\times\left(\frac{5}{19}\right)^{6}\times\left(\frac{5}{19}\right)^{-16}=\left(\frac{5}{19}\right)^{16}\times\left(\frac{5}{19}\right)^{-16}=\left(\frac{5}{19}\right)^{0}=1$$

(iii) $$\left[\left(\frac{18}{11}\right)^{3}\div\left(\frac{18}{11}\right)^{2}\right]^{5}\div\left[\left(\frac{18}{11}\right)^{2}\right]^{2}$$

$$=\left[\left(\frac{18}{11}\right)^{3}\times\left(\frac{18}{11}\right)^{-2}\right]^{5}\div\left(\frac{18}{11}\right)^{4}=\left(\frac{18}{11}\right)^{5}\times\left(\frac{18}{11}\right)^{-4}=\frac{18}{11}$$

(iv) $$\left[\left(\frac{-4}{9}\right)^{2}\right]^{8}\div\left[\left(\frac{-4}{9}\right)^{3}\right]^{5}\times\left(\frac{-4}{9}\right)$$

$$=\left(\frac{-4}{9}\right)^{16}\div\left(\frac{-4}{9}\right)^{15}\times\left(\frac{-4}{9}\right)=\left(\frac{-4}{9}\right)^{16}\times\left(\frac{-4}{9}\right)^{-15}\times\left(\frac{-4}{9}\right)=\left(\frac{-4}{9}\right)^{2}=\frac{16}{81}$$

(v) $$\left[\left(\frac{1}{10}\right)^{3}\right]^{2}\times\left[\left(\frac{1}{10}\right)^{6}\right]^{3}\div\left(\frac{1}{10}\right)^{25}$$

$$=\left(\frac{1}{10}\right)^{6}\times\left(\frac{1}{10}\right)^{18}\times\left(\frac{1}{10}\right)^{-25}=\left(\frac{1}{10}\right)^{24}\times\left(\frac{1}{10}\right)^{-25}=\left(\frac{1}{10}\right)^{-1}=10$$