

CHAPTER-6

DIRECT & INVERSE VARIATION

Students Learning Outcomes

After studying this chapter, students will be able to:

- Define continued ratio and recall direct and inverse proportion.
- Solve real life problems (involving direct and inverse proportion) using unitary method and proportion method.
- Solve real life problems related to time and work using proportion.
- Find relation between time and distance (i.e. speed).
- Convert chapters of speed (kilometer per hour into meter per second and vice versa).
- Solve variation related problems involving time and distance.

SOLVED EXERCISE 6.1

1. If $a:b = 3:5$ and $b:c = 5:6$ then find $a:b:c$

Solution:

We observe that b is the common member

$$\begin{array}{rcl}
 a & & b & & c \\
 3 & : & 5 & & \\
 \hline
 & & 5 & : & 6 \\
 \hline
 15 & : & 25 & : & 30
 \end{array}$$

$$3 : 5 : 6$$

$$\text{Thus } a:b:c = 3:5:6$$

2. If $r:s = 1:4$ and $s:t = 2:3$ then find $r:s:t$

Solution:

We observe that s is the common member

$$\begin{array}{rcl}
 r & & s & & t \\
 1 & : & 4 & & \\
 \hline
 & & 2 & : & 3 \\
 \hline
 2 & : & 8 & : & 12
 \end{array}$$

$$1 : 4 : 6$$

$$\text{Thus } r:s:t = 1:4:6$$

3. If $p:q = 1:2$ and $q:r = 1:2$ then find $p:q:r$.

Solution:

We observe that q is the common member

$$\begin{array}{rcl} p & & q & & r \\ 1 & : & 2 & & \\ \hline & & 1 & : & 2 \\ \hline 1 & : & 2 & : & 4 \end{array}$$

Thus $p:q:r = 1:2:4$

4. If $x:z = 3:2$ and $y:z = 1:2$ then find $x:y:z$.

Solution:

We observe that z is the common member

$$\begin{array}{rcl} x & & z & & y \\ 3 & : & 2 & & \\ \hline & & 2 & : & 1 \\ \hline 6 & : & 4 & : & 2 \end{array}$$

$$3 : 2 : 1$$

Thus $x:y:z = 3:1:2$

5. If $\lambda:m = 1:7$ and $\lambda:n = 5:6$ then find $\lambda:m:n$

Solution:

We observe that λ is the common ratio

$$\begin{array}{rcl} m & & \lambda & & n \\ 7 & : & 1 & & \\ \hline & & 5 & : & 6 \\ \hline 35 & : & 5 & : & 6 \end{array}$$

Thus $\lambda:m:n$ is $5:35:6$

6. In a bakery the ratio of the sale of bread to eggs is $2:3$ and the sale of eggs to milk is $3:1$ find the continued ratio of bread eggs & milk.

Solution:

$$\begin{array}{rcl} \text{bread} & \text{eggs} & \text{milk} \\ 2 & : & 3 \\ \hline & 3 & : & 1 \\ \hline 6 & : & 9 & : & 3 \\ 2 & : & 3 & : & 1 \end{array}$$

Thus bread : eggs : milk = $2:3:1$

7. Ahmad and Irfan got profit in a business in the ratio of $5:4$ and Irfan and waseem got a profit in the ratio of $8:9$. Find the ratio of profit among Ahmad Irfan and waseem.

Solution:

$$\begin{array}{rcl} \text{Ahmad} & & \text{Irfan} & & \text{Waseem} \\ 5 & & & & 4 \\ \hline & & 8 & : & 9 \end{array}$$

$$\begin{array}{rclcl} 40 & : & 32 & : & 36 \\ 10 & : & 8 & : & 9 \end{array}$$

Thus Ahmad : Irfan : Waseem = 10:8:9

8. According to a survey the people's liking for chicken and mutton are in the ratio of 2:1 and the people's liking for chicken and beef is in the ratio of 5:2. Find the ratio among people's liking for chicken, mutton, and beef.

Solution:

Mutton	Chicken	Beef
1	:	2
		5
<hr style="width: 100%;"/>		
5	:	10
		4

Thus chicken : mutton : beef = 10:5:4

9. In a maths test Zara, Moona and Komal got marks in the ratio as given below:

Zara to Moona = 4:5

Moona to Komal = 4:3

Find the continued ratio of marks obtained by Zara, Moona and Komal.

Solution:

Zara	Moona	Komal
4	:	5
		4
<hr style="width: 100%;"/>		
16	:	20
		15

Thus Zara : Moona : Komal = 16:20:15

SOLVED EXERCISE 6.2

1. Find the value of m in the following proportion.

(i) $13 : 3 = m : 6$

$$=$$

$$78 = 3m \Rightarrow m = 26$$

(ii) $m : 5 = 3 : 10$

$$= \Rightarrow 10m = 15$$

$$m = \Rightarrow m = 3/2$$

(iii) $35 : 21 = 5 : m$

$$= \Rightarrow 35m = 105$$

$$m = \Rightarrow m = 3$$

(iv) $9 : m = 54 : 42$

$$= \Rightarrow 54m = 378$$

$$m = 7$$

(v) $0.21 : 6.3 = 0.06 : m$

$$0.21m = (0.06)(6.3)$$

$$m = \Rightarrow m = 1.8$$

(vi) $1.1 : m = 0.55 : 0.27$

$$= \Rightarrow 0.55m = 1.1 \times 0.27$$

$$m = \Rightarrow m = 0.54$$

2. What is the fourth proportional of 2, 5 and 6.

Solution: Let x be the fourth proportion

$$2 : 5 :: 6 : x$$

$$2x = 30$$

$$x = 15$$

3. Find mean proportional of 4 and 16.

Solution: Let x be the mean proportion

$$4 : x :: x : 16$$

$$x^2 = 64 \Rightarrow x = 16$$

4. A worker is paid Rs. 2130 for 6 days. If his total wage during a month is Rs. 9230. Find the number of days he worked in the month.

Solution:

Days	Rupees
6	2130
↑ x	9230 ↑

$$\frac{6}{x} = \frac{2130}{9230}$$

$$(6)(9230) = 2130x$$

$$\frac{(6)(9230)}{2130} = x$$

$$x = 26 \text{ days}$$

5. Uzair takes 75 steps to cover a distance of 50m. How much distance will be covered in 375 steps.

Solution:

Steps	Distance
75	50
↑ 375	x ↑

$$\frac{75}{375} = \frac{50}{x}$$

$$\frac{(375)(50)}{75} = x$$

$$x = 250 \text{ m}$$

6. If 2 boxes occupy a space of 500cm³. Then how much space will be required for such 175 boxes.

Solution:

Boxes	Space
2	500
↑	↑
x	=
	= 43750cm ³

7. An army camp of 200 men has enough food for 60 days. How long will the food last if:

- (a) The number of men is reduced to 160.
(b) The number of men is increased to 240?

Solution:

(a)	Men	Days
↑	↓	
	=	
x	=	= 75days
(b)	Men	Days
↓	↑	
	=	
x	=	= 50days

SOLVED EXERCISE 6.3

1. If a man weaves 450cm cloth in 6 hours. How many meters of cloth can we weave in 14 hours?

Solution:

Cloth	Hours	
↑	↑	
=		
x	=	= 1050m

2. If a 162 km long road can be constructed in 9 months. Then the number of months required to construct a 306km long road.

Solution:

Road	Month	
↑	↑	
=		
x	=	= 17days

3. 540 men can construct a building in 7months. How many men should be removed from work to finish the building in 9 months.

Solution:

Men	Month	
↑	↓	
=		
x	=	= 420men

4. Asma can iron 5 shirts in 14 minutes. How long will it take to iron 35 shirts?

Solution:

Shirts	Minutes
5	14
↑ 35	x ↑

$$\frac{5}{35} = \frac{14}{x}$$

$$x = \frac{(14)(35)}{5} = 98 \text{ minutes}$$

$$x = 1 \text{ hour } 38 \text{ min}$$

5. 12 water pumps can empty a water tank in 20 minutes. But 2 pumps are out of order.

How long will the remaining pumps take to empty the tank.

Solution:

Water pumps	Minutes
12	20
↑ 10	x ↓

$$\frac{12}{10} = \frac{x}{20}$$

$$(20)(12) = x$$

$$\frac{240}{10} = x$$

$$X = 24 \text{ min.}$$

6. 14 horses graze a field in 25 days. In how many days will 35 horses graze it.

Solution:

Horses	Days
14	25
↑ 35	x ↓

$$\frac{14}{35} = \frac{x}{25}$$

$$(14)(25) = x$$

$$\frac{350}{35} = x$$

$$X = 10 \text{ days}$$

7. A mason can repair a 744m long tracks in 24 days. If he repairs 589m track. Then find how many days he will take to repair the remaining track.

Solution: Remaining track = 744 - 589 = 155

Tracks Days

744	24
↑ 155	x ↑

$$\frac{744}{155} = \frac{24}{x}$$

$$(155)(24) = x$$

$$\frac{3720}{744} = x$$

$$X = 5 \text{ days}$$

8. A farmer can plough an area of 40 aces in 16 hours. How aces will be plough in 36 hours.

Solution:

Aces	Hours
40	16
↑ x	36 ↑

$$\frac{40}{x} = \frac{16}{36}$$

$$(40)(36) = x$$

$$16$$

$$X = 90 \text{ days}$$

9. A dishwasher cleans 1350 dishes in 1 hour. How many dishes will it wash in 16 more minutes?

Solution: 1 hour = 60 minutes

Dishes	Minutes
1350	60
$\uparrow x$	16 \uparrow
$\frac{1350}{x}$	$\frac{60}{16}$
$(1350)(16) = x$	
60	
$X = 1710 \text{ dishes}$	

SOLVED EXERCISE 6.4

1. Convert the unit of speed into meters per second.

Solution:

(i) 72 km/hour

Speed = $\frac{72 \times 1000}{3600} = 20 \text{ m/s}$

(ii) 144 km/hour

Speed = $\frac{144 \times 1000}{3600} = 40 \text{ m/s}$

(iii) 216 km/hour

Speed = $\frac{216 \times 1000}{3600} = 60 \text{ m/s}$

(iv) 360 km/hour

Speed = $\frac{360 \times 1000}{3600} = 100 \text{ m/s}$

(v) 180 km/hour

Speed = $\frac{180 \times 1000}{3600} = 50 \text{ m/s}$

(vi) 1152 km/hour

Speed = $\frac{1152 \times 1000}{60 \times 60} = 320 \text{ m/s}$

2. Convert the unit of speed into kilometers per hour.

Solution:

(i) 10 m/sec

Speed = $\frac{10 \times 3600}{1000} = 36 \text{ km/hour}$

(ii) 25 m/sec

Speed = $\frac{25 \times 3600}{1000} = 90 \text{ km/hour}$

(iii) 5 m/sec

Speed = $\frac{5 \times 3600}{1000} = 18 \text{ km/hour}$

(iv) 15 m/sec

Speed = $\frac{15 \times 3600}{1000} = 54 \text{ km/hour}$

(v) 30 m/sec

Speed = $\frac{30 \times 3600}{1000} = 108 \text{ km/hour}$

(vi) 20 m/sec

Speed = $\frac{20 \times 3600}{1000} = 72 \text{ km/hour}$

3. Iram walks up to her school at a speed of 4 km/hour. It takes 45 minutes to reach the school. How far is her school from her home?

Solution:

Speed = $4 \text{ km/hour} = \frac{4 \times 1000}{3600} \text{ m/s}$

Speed = 1.11 m/s

Time = 45 minutes = $45 \times 60 = 2700 \text{ sec}$

Distance = $1.11 \times 2700 = 3000 \text{ m}$

4. A train leaves Lahore at 4:00 PM and reaches Karachi at 10:00 a.m next day. The speed of the train was 70 km/hour. Find the distance between Lahore and Karachi.

Solution:

Time = 18 hours

Speed = 70 km/hours

Speed = 70 km/h =

Distance = $70 \times 18 = 1260 \text{ km}$

5. A cyclist crosses a 30 meter long bridge in 3 minutes. Find the speed of the cyclist.

Solution:

Distance = 30 meter

Time = 3 min = $3 \times 60 = 180 \text{ sec}$

Speed = $\frac{d}{t} = \frac{30}{180} = \frac{1}{6} \text{ m/s}$

6. A car covers 201 kilometers in 3 hours. How much distance will it cover in 7 hour.

Solution:

Distance = 201 kilometer

Time = 3 hours
 Speed = 67 kilometer/hour
 Distance covers in 7 hour = $67 \times 7 = 469 \text{ km}$

7. A truck moves at the speed of 36 kilometers per hour. How far will it travel in 15 seconds?

Solution:

Speed = $36 \text{ km/h} = 10 \text{ m/s}$

Time = 15 sec

Distance = $10 \times 15 = 150 \text{ m}$

8. A bus leaves Islamabad at 11:00 a.m and reaches Lahore at 3:00 p.m. If the distance between Lahore and Islamabad is 380 km. Find the speed of the bus.

Solution:

Time = 4 hours

Distance = 380 km

Speed = ?

Speed = $\frac{380 \text{ km}}{4 \text{ h}}$

= 95 km/hour

SOLVED REVIEW EXERCISE 6

1. Answer the following questions.

- (i) Define direct proportion.

Answer:

A relation in which one quantity increases or decreases in the same proportion by increasing or decreasing the other quantity, is called the direct proportion.

- (ii) What is the continued ratio?

Answer:

Two ratios of three quantities can be combined into a continued ratio to express the relation of these quantities. The final combination is called the continued ratio.

- (iii) Write the formula to show the relation between time, speed and distance.

Answer:

Distance = speed \times time or speed = distance \div time

- (iv) Define speed.

Answer:

The distance covered per unit of time is called speed.

2. Fill in the blanks.

- (i) Distance is directly proportional to the time and speed.
 (ii) Number "of workers is inversely proportional to the time ratio.
 (iii) The combination of two ratios of three quantities is called a continued ratio.
 (iv) Distance = speed \times time
 (v) Speed = .
 (vi) In two ratios $a:b$ and $b:c$, b is called the common ratio.

3. Tick (✓) the correct answer.

- (i) When we convert 36 km/hr into m/sec, we get:
 (a) 1 m/sec (b) 5 m/sec

- (c) ✓ 10m/sec (d) 20m/sec
 (ii) An interval between two happenings is called:
 (a) ✓ time (b) speed
 (c) distance (d) ratio
 (iii) If, $2 : 5 = x : 20$, then $x =$ _____
 (a) 50 (b) 2 (c) _____ (d) ✓ 8
 (iv) A man is walking at the speed of 1km/hr. In 30 minutes, he will walk:
 (a) 500 km (b) 0.5m (c) ✓ 500m (d) 0.05 km

4. Find the missing terms in the table, if x is directly proportional to the y .

			6	8	10
x	2	4		16	<u>20</u>
y	4	<u>8</u>	12		

5. In a class, 8 ice creams are served for every group of 5 students. How many ice-creams will be served if there are 40 students in the class.

Solution:

Ice creams	Students
x	5
↑ x	40 ↑
$\frac{8}{x}$	$\frac{5}{40}$
$\frac{(8)(5)}{40} = x$	
$X = 64$ stu	

6. In a hostel of 50 girls there are food provisions for 40 days. If 30 more girls join the hostel, how long will the provisions last.

Solution:

Girls	Days
50	40
↑ 30	x ↓
$\frac{50}{30}$	$\frac{x}{40}$
$\frac{(50)(40)}{40} = x$	
$X = 25$ days	

7. How many Days will 1648 Persons take to construct a bridge if 721 persons can build the same in 48 days.

Solution:

Persons	Days
1648	x
↓ 721	48 ↑

$$\frac{1648}{721} = \frac{48}{x}$$

$$(721)(48) = x$$

$$1648$$

$$x = 21 \text{ days}$$

8. A rickshaw travels at the speed of 36 km per hour. How much distance will it travel in 20 second.

Solution:

$$\begin{aligned} \text{Speed} &= 36 \text{ km/hour} \\ &= 36000/3600 = 10\text{m/sec} \\ \text{Time} &= 20 \text{ second} \\ \text{Speed} &= \\ 10 &= \frac{\text{Distance}}{20} \end{aligned}$$

$$\text{Distance} = 10 \times 20 = 200\text{m}$$

9. A bus covers a distance in 3 hours at the speed of 60km/ hour. How much time will it take to cover the same distance at a speed of 80km/hour.

Solution:

$$\begin{aligned} \text{Speed} &= \\ 60\text{km/h} &= \frac{\text{Distance}}{3\text{h}} \\ \text{Distance} &= 180\text{km} \\ \text{Speed} &= \\ 80\text{km/h} &= \end{aligned}$$

Time = 2 hours.

This question can also be solved by ratio and proportion method, as under

Speed	Time
60	3
80	x
=	
x = 2 hours	