CHAPTER-8
ALGEBRAIC EXPRESSIONS

Students Learning Outcomes

After studying this chapter, students will be able to:

- Define a constant as a symbol having a fixed numerical value.
- Recall a variable as a quantity which can take various numerical values.
- Recall a literal as an unknown number represented by a letter of an alphabet.
- Recall an algebraic expression as a combination, of constants and variables connected by the sign of fundamental operations.
- Define a polynomial as an algebraic expression in which the powers of variables are all whole numbers.
- Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms and. three terms respectively.
- Add two or more polynomials.
- Subtract a polynomial from another polynomial.
- Find the product of:
  - monomial with monomial.
  - monomial with binomial/trinomial.
  - binomials with binomial/trinomial.
- Simplify algebraic expressions involving additions, subtraction and multiplication.
- Recognize and verify the algebraic identities:
  - $(x + a)(x + b) = x^2 + (a + b)x + ab$,
  - $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$,
  - $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$,
  - $a^2 - b^2 = (a - b)(a + b)$.
- Factorize an algebraic expression (using algebraic identities).
- Factorize an algebraic expression (making groups).

SOLVED EXERCISE 8.1

1. Add the terms to write an algebraic expression.

Solution:

(i) $2ab, 3bc, ca$
$= 2ab + 3bc + ca$

(ii) $7t^2, 3m^2, -8$
$= 7t^2 + 3m^2 + (-8) \quad \Rightarrow 7t^2 + 3m^2 - 8$

(iii) $p^2, -q^2, -r^2$
$= p^2 - q^2 - r^2$

(iv) $5xyz, 2yz, -8xy$
$= 5xyz + 2yz - 8xy$

(v) $-2ab, -2a - bc, c$
$= -2ab - 2a - bc + c$

(vi) $9lm, 8mn, -10ml, -2$
$= 9lm + 8mn - 10ml - 2$

2. Write constants and variables used in each expression.

Solution:

		Variables	Constants
(i)	$x+3$	x	3
(ii)	$3a+b-2$	a, b	-2
(iii)	$l^2 + m^2 + n^2$	l, m, n	
(iv)	$5a$	a	
(v)	$2x^2-1$	x	-1
(vi)	$3l^2-4n^2$	l, n	

3. Identify monomials, binomials and trinomials.

Solution:

(i)	$x+y-z$	trinomial
(ii)	-61	monomial
(iii)	$2x^2-3$	binomial
(iv)	abc	monomial
(v)	$x^2+2xy+y^2$	trinomial
(vi)	$(-a)^3$	monomial
(vii)	$l-m$	binomial
viii)	$7a^2 - b^2$	binomial
(ix)	$lm + mn + nl$	trinomial
(x)	$2a-3b-4c$	trinomial
(xi)	$11x^2y^2$	monomial
(xii)	$a^3 + a^2b + ab^2$	trinomial

SOLVED EXERCISE 8.2

1. Add the following polynomials.

(i) $x^2 + 2xy + y^2, x^2 - 2xy + y^2$

Solution:

$x^2 + 2xy + y^2,$

$\underline{x^2 - 2xy + y^2}$

$\underline{2x^2 + 2y^2 = 2(x^2 + y^2)}$

(ii) $x^3 + 3x3x2\text{-}2xy^2 + y^3, 2x^3 - 5x^2y - 3xy^2 - 2y^3$

Solution:

$x^3 + 3x^2y - 2xy^2 + y^3$

$\underline{2x^3 - 5x^2y - 3xy^2 - 2y^3}$

$\underline{3x^3 - 2x^2y - 5xy^2 - y^3}$

(iii) $a^5 + a^3b - 2ab^3 + b^3, 4a^5 + 3a^3b + 2ab^3 + 5b^3$

Solution:

$a^5 + a^3b - 2ab^3 + b^3$

$\underline{4a^5 + 3a^3b + 2ab^3 + 5b^3}$

$\underline{5a^5 + 4a^3b + 6b^3}$

(iv) $2x4y\text{-} 4x^3y^2 + 33x2y3\text{-}7xy^4, x^4y - 4x^3y^2 - 3x^2y^3 + 8xy^4$

Solution:

$2x^4y - 4x^3y^2 + 3x^2y^3 - 7xy^4$

$\underline{x^4y - 4x^3y^2 - 3x^2y^3 + 8xy^4}$

$\underline{3x^4y - 8x^3y^2 + xy^4}$

(v) $ab^5 + 12a^2b^4 - 6a^3b^3 + 10a^4b^2 - a^5b$, $4ab^5 - 8a^2b^4 + 6a^3b^3 - 6a^4b^2 + 4a^5b$

Solution:

$ab^5 + 12a^2b^4 - 6a^3b^3 + 10a^4b^2 - a^5b,$

$\underline{4ab^5 - 8a^2b^4 + 6a^3b^3 - 6a^4b^2 + 4a^5b}$

$\underline{5ab^5 + 4a^2b^4 + 4a^4b^2 + 3a^5b}$

2. If A= x-2y + z, B = -2x + y + z and C = x+y-2z then find.

Solution:

(i) $A - B = (x - 2y + z) - (-2x + y + z)$

$= x - 2y + z + 2x - y - z = 3x\text{-}3y = 3(x - y)$

(ii) $B - C = (-2x + y + z) - (x + y - 2z)$

$= -2x + y + z - x - y + 2z = -3x + 3z = 3(z - x)$

(iii) $C - A = (x + y - 2z) - (x - 2y + z)$
$= x + y - 2z - x + 2y - z = 3y - 3z = 3(y - z)$

(iv) $A - B - C = (x - 2y + z) - (-2x + y + z) - (x + y - 2z)$
$= x - 2y + z + 2x - y - z - x - y + 2z = 2x - 4y + 2z = 2(x - 2y + z)$

(v) $A + B - C = (x - 2y + z) + (-2x + y + z) - (x + y - 2z)$
$= x - 2y + z - 2x + y + z - x - y + 2z$
$= -2x - 2y + 4z = -2(x + y - 2z)$

(vi) $A - B + C = (x - 2y + z) - (-2x + y + z) + (x + y - 2z)$
$= x - 2y + z + 2x - y - z + x + y - 2z = 4x - 2y - 2z = 2(2x - y - z)$

3. What must be added to $x^7 - x^6 + x^5 - x^4 + x^3 - x^2 + x + 1$ to get $x^7 + x^5 + x^3 - 1$?

Solution:

$$\begin{array}{l} x^7 + x^5 + x^3 - 1 \\ \underline{\pm x^7 \pm x^5 \pm x^3 \pm 1 \mp x^6 \mp x^4 \mp x^2 \pm x} \\ \qquad -2 + x^6 + x^4 + x^2 - x \end{array}$$

Added by $x^6 + x^4 + x^2 - x - 2$ then we get $x^7 + x^5 + x^3 - 1$

4. What must be added to $2x^4y^3 - x^3y^2 - 3x^2y - 4$ to get $5x^4y^3 + 2x^3y^2 + x^2y - 9$?

Solution:

$$\begin{array}{l} 5x^4y^3 + 2x^3y^2 + x^2y - 9 \\ \underline{\pm 2x^4y^3 \mp x^3y^2 \mp 3x^2y \mp 4} \\ 3x^4y^3 + 3x^3y^2 + 4x^2y - 5 \end{array}$$

Added by $3x^4y^3 + 3x^3y^2 + 4x^2y - 5$ then we get $5x^4y^3 + 2x^3y^2 + x^2y - 9$

5. What must be subtracted from $5x^5y^5 - 3x^3y^3 + 10xy - 9$ to get $3x^5y^5 + 7x^3y^3 - 11xy + 19$

Solution:

$$\begin{array}{l} 5x^5y^5 - 3x^3y^3 + 10xy - 9 \\ \underline{\pm 3x^5y^5 \pm 7x^3y^3 \mp 11xy \pm 19} \\ 2x^5y^5 - 10x^3y^3 + 21xy - 28 \end{array}$$

Subtracted by $2x^5y^5 - 10x^3y^3 + 21xy - 28$ then we get $3x^5y^5 + 7x^3y^3 - 11xy + 19$.

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SOLVED EXERCISE 8.3

1. Multiply

Solution:

(i) $7m$ and -8

$(7m) \times (-8) = -56m$

(ii) $2ab$ and $3a^2b^2$

$= (2ab) \times (3a^2b^2)$

$= 6a^3b^3$

(iii) $4xy$ and $2x^2y$

$= (4xy) \times (2x^2y)$

$= 8x^3y^2$

(iv) -4ab aand-2bc

$= (-4ab) \times (-2bc)$

$= 8ab^2c$

(v) $3\lambda m^3$ and $3mn$

$= (3\lambda m^3) \times (3mn) = 9\lambda m^4n$

(vi) $-6x^2y$ and $3xyz^2$

$= (-6x^2y) \times (3xyz^2)$

$= -18x^3y^2z^2$

(vii) $(2a^2b)$ and $5a^2b^3$

$(2a^2b) \times (5a^2b^3) = 10\ a^4b^4$

(viii) λ^2mn and λm^3n^6

$(\lambda^2mn) \times (\lambda m^3n^6) = /^3m^4n^7$

(ix) $-4x^2yz^7$ and $8xy^4z^3$

$= (-4x^2yz^7) \times (8xy^4z^3) = -32x^3y^5z^{10}$

2. Simplify:

Solution:

(i) $\lambda m(\lambda+m) = \lambda^2m + \lambda m^2$

(ii) $4p(p+q) = 4p^2+4pq$

(iii) $3a(a-b) = 3a^2-3ab$

(iv) $2x(3x+4y) = 6x^2+8xy$
(v) $2a(2b - 2c) = 4ab - 4ac$
(vi) $2\lambda m(\lambda^2 m^2 - n) = 2\lambda^3 m^3 - 2\lambda mn$
(vii) $a(a + b - c) = a^2 + ab - ac$
(viii) $3x(x - 2y - 2z) = 3x^2 - 6xy - 6xz$
(ix) $3p^2q(p^3 + q^2 - r^4) = 3p^5q + 3p^2q^3 - 3p^2qr^4$

SOLVED EXERCISE 8.4

1. **Multiply**

Solution:

(i) (3a+4) (2a-l) $3a + 4$ $\times\ 2a - 1$ $6a^2 + 8a$ $-3a - 4$ $6a^2 + 5a - 4$	(ii) (m+2) (m-2) $m + 2$ $\times\ m - 2$ $m^2 + 2m$ $-2m - 4$ $m^2 - 4$
(iii) $(x-1)(x^2+x+1)$ $x^2 + x + 1$ $\times\ x - 1$ $x^3 + x^2 + x$ $-x^2 - x - 1$ $x^3 - 1$	(iv) $(p - q)(p^2 + pq + q^2)$ $p^2 + pq + q^2$ $\times\ p - q$ $p^3 + p^2q + pq^2$ $-p^2q - pq^2 - q^3$ $p^3 - q^3$
(v) $(x + y)(x^2 - xy + y^2)$ $x^2 - xy + y^2$ $\times\ x + y$ $x^3 - x^2y + xy^2$ $+x^2y - xy^2 + y^3$ $x^3 + y^3$	(vi) $(a + b)(a - b)$ $a + b$ $\times\ a - b$ $a^2 + ab$ $-ab - b^2$ $a^2 - b^2$

(vii) $(\lambda - m)(\lambda^2 - 2\lambda m + m^3)$

$\lambda^2 - 2\lambda m + m^3$
$\times\ \lambda - m$
$\lambda^3 - 2\lambda^2 m + \lambda m^3$
$-\lambda^2 m \quad + 2\lambda m^2 - m^4$
$\lambda^3 - 3\lambda^2 m + \lambda m^3 + 2\lambda m^2 - m^4$

(ix) $(3p - 4q)(3p + 4q)$

$3p - 4q$
$\times\ 3p + 4q$
$9p^2 - 12pq$
$+12pq - 16q^2$
$9p^2 - 16q^2$

(xi) $(1 - 2c)(1 + 2c)$

$1 - 2c$
$\times\ 1 + 2c$
$1 - 2c$

$+ 2c - 4c^2$

$1 - 4c^2$

(viii) $(2x - 1)(4x^2 + 2x + 1)$

$4x^2 + 2x + 1$

$\times\ 2x - 1$

$8x^3 + 4x^2 + 2x$

$-4x^2 - 2x - 1$

$8x^3 - 1$

x) $(a + b)(a^2 - ab + b^2)$

$a^2 - ab + b^2$

$\times\ a + b$

$a^3 - a^2b + ab^2$

$+ a^2b - ab^2 + b^3$

$a^3 + b^3$

(xii) $(3 - b)(2b - b^2 + 3)$

$2b - b^2 + 3$

$\times\ 3 - b$

$6b - 3b^2 + 9$

$-3b - 2b^2 \quad + b^3$

$3b - 5b^2 + 9 + b^3$

2. Simplify:

Solution:

(i) $(x^2 + y^2)(3x + 2y) + xy(x - 3y)$

$= 3x^3 + 2x^2y + 3xy^2 + 2y^3 + x^2y - 3xy^2$

$= 3x^3 + 3x^2y + 2y^3$

(ii) $(4x + 3y)(2x - y) - (3x - 2y)(x + y)$

$= (8x^2 - 4xy + 6xy - 3y^2) - (3x^2 + 3xy - 2xy - 2y^2$

$= 8x^2 + 2xy - 3y^2 - 3x^2 - xy + 2y^2$

$= 5x^2 + xy - y^2$

(iii) $(2m^2 - 5m + 4)(m + 2) - (m^2 + 7m - 8)(2m - 3)$

$= (2m^3 - 5m^2 + 4m + 4m^2 - 10m + 8) - (2m^3 + 14m^2 - 16m - 3m^2 - 21m + 24)$

$= (2m^3 - m^2 - 6m + 8) - (2m^3 + 11m^2 - 37m + 24)$

$= 2m^3 - m^2 - 6m + 8 - 2m^3 - 11m^2 + 37m - 24$

$= -12m^2 + 31m - 16$

(iv) $(3x^2 + 2xy - 2y^2)(x + y) - (x^2 - xy + y^2)(x - y)$

$= (3x^3 + 2x^2y - 2xy^2 + 3x^2y + 2xy^2 - 2y^3) - (x^3 - x^2y + xy^2 - x^2y + xy^2 - y^3)$

$= 3x^3 + 5x^2y - 2y^3 - x^3 + 2x^2y - 2xy^2 + y^3$

$= 2x^3 + 7x^2y - 2xy^2 - y^3$

SOLVED EXERCISE 8.5

1. **Simplify the following binomials.**

Solution:

(i) $(x+1)(x+2)$

$= x^2 + 2x + x + 2 = x^2 + 3x + 2$

(ii) $(x-2)(x-4)$

$= x^2 - 4x - 2x + 8 = x^2 - 6x + 8$

(iii) $(a+5)(a+3)$

$= a^2 + 3a + 5a + 15 = a^2 + 8a + 15$

(iv) $(b+6)(b-9)$

$= a - 9b + 6b - 54 = b^2 - 3b - 54$

(v) $(2x+3)(2x-7)$

$= 4x^2 - 14x + 6x - 21 = 4x^2 - 8x - 21$

(vi) $(2y+1)(2y+5)$

$= 4y^2 + 10y + 2y + 5 = 4y^2 + 12y + 5$

(vii) $(3b-1)(3b-7)$

$= 9b^2 - 21b - 3b + 7 = 9b^2 - 24b + 7$

(viii) $(4x+5)(4x+3)$

$= 16x^2 + 20x + 12x + 15$

$= 16x^2 + 32x + 15$

(ix) $(5y-2)(5y+6)$

$= 25y^2 + 30y - 10y - 12 = 25y^2 + 20y - 12$

(x) $(8a+7)(8a-3)$

$= 64a^2 - 24a + 56a - 21$

$= 64a^2 + 32a - 21$

2. **By using identity, find the square of the following binomials.**

Solution:

(i) $(x+y)^2 = x^2 + y^2 + 2xy$

(ii) $(3a+4)^2 = (3a)^2 + (4)^2 + 2(3a)(4)$

$= 9a^2 + 24a + 16$

(iii) $(x - y)^2 = x^2 - 2(x)(y) + y^2$
$= x^2 - 2xy + y^2$

(iv) $(a + 2b)^2 = (a)^2 + 2(a)(2b) + (2b)^2$
$= a^2 + 4ab + 4b^2$

(v) $(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$
$= 4x^2 + 12xy + 9y^2$

(vi) $(2a-b)^2 = (2a)^2 - 2(2a)(b) + (b)^2$
$= 4a^2 - 4ab + b^2$

(vii) $(3x-2y)^2 = (3x)^2 - 2(3x)(2y) + (2y)^2$
$= 9x^2 - 12xy + 4y^2$

(viii) $(4x + 5y)^2 = (4x)^2 + 2(5y)(4x) + (5y)^2$
$= 16x^2 + 40xy + 25y^2$

(ix) $(7a-8b)^2$
$= (7a)^2 - 2(7a)(8b) + (8b)^2$
$= 49a^2 - 112ab + 64b^2$

3. **Find the product of the following binomials by using formula.**

Solution:

(i) $(x+y)(x-y) = (x)^2 - (y)^2 = x^2 - y^2$

(ii) $(3a-8)(3a+8) = (3a)^2 - (8)^2 = 9a^2 - 64$

(iii) $(2a + 7b)(x- 7b) = (2a)^2 - (7b)^2 = 4a^2 - 49b^2$

(iv) $(x+3y)(x-3y) = (x)^2 - (3y)^2 = x^2 - 9y^2$

(v) $(6a - 5b)(6a + 5b) = (6a)^2 - (5b)^2 = 36a^2 - 25b^2$

(vi) $(9x - 1ly)(9x + 11y)$
$= (9x)2 - (11y)^2$
$= 81x^2 - 121y^2$

SOLVED EXERCISE 8.6

1. Resolve into factors.

Solution:

(i) $5x^2y - 10xy^2$
$= 5xy(x - 2y)$

(ii) $2a - 4b + 6c$
$= 2(a - 2b + 3c)$

(iii) $9x^4 + 6y^2 + 3$
$= 3(3x^4 + 2y^2 + 1)$

(iv) $a^3b + a^2b^2 + ab^3$
$= ab(a^2 + ab + b^2)$

(v) $x^2yz + xy^2z + xyz^2$
$= xyz(x+y+z)$

(vi) $bx^3 + bx^2 - x - 1$
$= bx^2(x+1) - 1(x+1)$
$= (x + 1)(bx^2 - 1)$

(vii) $x^2 + qx + px + pq$
$= x(x+q) + p(x+q) \Rightarrow (x + q)(x + p)$

(viii) $ab - a - b + l$
$= a(b - 1) - 1(b - 1)$
$= (b - 1)(a - 1)$

(ix) $(pm + n) + (pn + m)$
$= pm + m + pn + n$
$= m(p+1) + n(p+1)$
$= (p+1)(m+n)$

(x) $(a^2 + bc) - (b + c)a$
$= a^2 + bc - ab - ac$
$= a^2 - ac - ab + bc$
$= a(a - c) - b(a - c)$
$= (a - b)(a - c)$

(xi) $x^2 - (m + n)x + mn$
$= x^2 - mx - nx + mn$
$= x(x - m) - n(x - m)$
$= (x - m)(x - n)$

(xii) $x^3 - y^2 + x - x^2y^2$
$= x^3 - x^2y^2 + x - y^2$
$= x^2(x - y^2) + 1(x - y^2)$
$= (x - y^2)(x^2 + 1)$

2. Factorize by using identity.

Solution:

(i) $4a^2 - 25$
$= (2a)^2 - (5)^2 = (2a + 5)(2a - 5)$

(ii) $4x^2 - 9y^2$
$= (2x)^2 - (3y)^2$
$= (2x + 3y)(2x - 3y)$

(iii) $9a^2 - b^2$
$= (3a)^2 - (b)^2 = (3a + b)(3a - b)$

(iv) $9m^2 - 16n^2$
$= (3m)^2 - (4n)^2 = (3m + 4n)(3m - 4n)$

(v) $16b^2 - a^2$
$= (4b)^2 - (a)^2 = (4b + a)(4b - a)$

(vi) $-1(1)^2 + (x+1)^2$
$= (x + 1)^2 - (1)^2$
$= (x + 1 - 1)(x + 1 + 1)$
$= x(x + 2)$

(vii) $8x^2 - 18y^2$
$= 2(4x^2 - 9y^2)$
$= 2[(2x)2-(3y)^2]$
$= 2(2x + 3y)(2x - 3y)$

(viii) $(a + b)^2 - (c)^2$
$= (a + b + c)(a + b - c)$

(ix) $x^2 - (y + z)^2$
$= (x + y + z)(x - y - z)$

(x) $7x^2-7y^2$
$= 7(x^2 - y^2)$
$= 7(x + y)(x - y)$

(xi) $5a^2 - 20b^2$
$= 5(a^2 - 4b^2)$
$= 5[(a)^2 - (2b)^2]$
$= 5(a + 2b)(a - 2b)$

(xii) x^4-y^4
$= (x^2)^2 - (y^2)^2$
$= (x^2 - y^2)(x^2 + y^2)$
$= (x + y)(x - y)(x^2 + y^2)$

SOLVED EXERCISE 8.7

1. Resolve into factors by using identity.

Solution:

(i) $x^2 + 8x+16$
$= (x)^2 + 2(x)(4) + (4)^2$
$= (x + 4)^2$

(ii) x^2-2x+l
$= (x)^2 - 2(x)(1) + (1)^2$
$= (x- 1)^2$

(iii) $a^4-14a^2 + 49$
$= (a^2)^2 - 2(a^2)(7) + (7)^2 = (a^2 - 7)^2$

(iv) $l +10m+25m^2$
$= (1)^2 + 2(1)(5m) + (5m)^2$
$= (1 + 5m)^2$

(v) $4x- 12xy + 9y^2$
$= (2x)^2 - 2(2x)(3y) + (3y)^2$
$= (2x - 3y)^2$

(vi) $9a^2 + 30ab + 25b^2$
$= (3a)^2 + 2(3a)(5b) + (5b)^2$
$= (3a + 5b)^2$

(vii) $16a^2 + 56ab + 49b^2$
$= (4a)^2 + 2(4a)(7b) + (7b)^2$
$= (4a + 7b)^2$

(viii) $36x^2 + 108xy + 8\,ly^2$
$= (6x)^2 + 2(6x)(9y) + (9y)^2$
$= (6x + 9y)^2$

(ix) $49m^2+154m+121$
$= (7m)^2 + 2(7m)(11) + (11)^2$
$= (7m + 11)^2$

(x) $64a^2-208ab + 169b^2$
$= (8a)^2 - 2(8a)(13b) + (13b)^2$
$= (8a -13b)^2$

(xi) $3x^4+24x^2+48$
$= 3(x^4 + 8x^2 +16)$
$= 3[(x^2)^2 + 2(x^2)(4) + (4)^2]$
$= 3(x^2+4)^2$

(xii) $11x^2 + 22x + 11$
$= 11(x^2 + 2x +1)$
$= 11[(x)^2+2(x)(1)+(1)^2]$
$= 11(x + 1)^2$

(xiii) $44a^4 - 44a^3b + 11a^2b^2$
$= 11[4a^4 - 4a^3b + a^2b^2]$
$= 11[(2a^2)^2 - 2(2a^2)(ab) + (ab)^2]$
$= 11(2a^2 - ab)^2$

(xiv) $a^4 + 16a^2b + 64b^2$
$= (a^2)^2 + 2(a^2)(8b) + (8b)^2$
$= (a^2 + 8b)^2$

(xv) $1 - 4xyz + 4x^2y^2z^2$
$= (1)^2 - 2(1)(2xyz) + (2xyz)^2$
$= (1 - 2xyz)^2$

(xvi) $16x^3y - 40x^2y^2 + 25xy^3$
$= xy[16x^2 - 40xy + 25y^2]$
$= xy[(4x)^2 - 2(4x)(5y) + (5y)^2]$
$= xy(4x - 5y)^2$

2. Factorize by using the identity.

(i) $a^2x^2 + 2abcx + b^2c^2$
$= (ax)^2 + 2(ax)(bc) + (bc)^2$
$= (ax + bc)^2$

(ii) $\frac{\ell^2}{4} + \ell mn + m^2n^2$

$$= \left(\frac{\ell}{2}\right)^2 + 2\left(\frac{\ell}{2}\right)(mn) + (mn)^2$$

$$= \left(\frac{\ell}{2} + mn\right)^2$$

(iii) $\frac{4}{9}x^2 - xy + \frac{9}{16}y^2$

$$= \left(\frac{2}{3}x\right)^2 - 2\left(\frac{2}{3}x\right)\left(\frac{3}{4}y\right) + \left(\frac{3}{4}y\right)^2$$

$$= \left(\frac{2}{3}x - \frac{3}{4}y\right)^2$$

(iv) $\frac{121}{169}a^2 - 2ab + \frac{169}{121}b^2$

$$= \left(\frac{11}{13}a\right)^2 - 2\left(\frac{11}{13}a\right)\left(\frac{13}{11}b\right) + \left(\frac{13}{11}b\right)^2$$

$$= \left(\frac{11}{13}a - \frac{13}{11}b\right)^2$$

(v) $\frac{a^2x^2}{b^2} - \frac{2axy}{c} + \frac{b^2y^2}{c^2}$

$$= \left(\frac{ax}{b}\right)^2 - 2\left(\frac{ax}{b}\right)\left(\frac{by}{c}\right) + \left(\frac{by}{c}\right)^2$$

$$= \left(\frac{ax}{b} - \frac{by}{c}\right)^2$$

(vi) $\frac{\ell^4}{n}x^4 - 2\frac{\ell^2m^2}{n}x^2y^2 + \frac{m^4}{n}y^4$

$$= \frac{1}{n}\left[\ell^4x^4 - 2\ell^2m^2x^2y^2 + m^4y^4\right]$$

$$= \frac{1}{n}\left[(\ell^2x^2)^2 - 2(\ell^2x^2)(m^2y^2) + (m^2y^2)^2\right]$$

$$= \frac{1}{n}(\ell^2x^2 - m^2y^2)^2$$

(vii) $a^2b^2c^2x^2 - 2a^2b^2cdxy + a^2b^2d^2y^2$

$$= (abcx)^2 - 2(abcx)(abdy) + (abdy)^2$$

$$= (abcx - abdy)^2$$

(viii) $\frac{b^2}{c^2}x^4 + \frac{2b}{a}x^3y + \frac{c^2}{a^2}x^2y^2$

$$= \left(\frac{b}{c}x^2\right)^2 + 2\left(\frac{b}{c}x^2\right)\left(\frac{c}{a}xy\right) + \left(\frac{c}{a}xy\right)^2$$

$$= \left(\frac{b}{x}x^2 + \frac{c}{a}xy\right)^2$$

SOLVED EXERCISE 8.8

1. Factorize the following expressions.

Solution:

(i) $\ell x - my + mx - \ell y$

$= \ell x - \ell y + mx - my$

$= \ell(x - y) + m(x - y)$

$= (x - y)(\ell - m)$

(ii) $2xy - 6yz + x - 3z$

$= 2xy + x - 6yz - 3z$

$= x(2y + 1) - 3z(2y + 1)$

$= (2y + 1)(x - 3z)$

(iii) $p^2 + 2p - 3p - 6$

$= p(p + 2) - 3(p + 2)$

$= (p + 2)(p - 3)$

(iv) $x^2 + 5x - 2x - 10$

$= x^2 + 5x - 2x - 10$

$= x(x + 5) - 2(x + 5)$

$= (x + 5)(x - 2)$

(v) $m^2 - 7m + 2m - 14$

$= m(m - 7) + 2(m - 7)$

$= (m - 7)(m + 2)$

(vi) $a^2 + 3a - 4a - 12$

$= a(a + 3) - 4(a + 3)$

$= (a + 3)(a - 4)$

(vii) $x^2 - 9x + 3x - 27$

$= x(x - 9) + 3(x - 9)$

$= (x - 9)(x + 3)$

(viii) $z^2 - 8z - 4z + 32$

$= z(z - 8) - 4(z - 8)$

$= (z - 8)(z - 4)$

(ix) $t^2 - st + t - s$

$= t(t - s) + 1(t - s)$

$= (t - s)(t + 1)$

(x) $n^2 + 5n - n - 5$

$= n(n + 5) - 1(n + 5)$

$= (n + 5)(n - 1)$

(xi) $a^2b^2 + 7ab - ab - 7$

$= ab(ab + 7) - 1(ab + 7)$

$= (ab + 7)(ab - 1)$

(xii) $\ell^2m^2 - 13\ell m - 2\ell m + 26$

$= \ell m(\ell m - 13) - 2(\ell m - 13)$

$= (\ell m - 13)(\ell m - 2)$

SOLVED REVIEW EXERCISE 8

1. Answer the following questions.

(i) What is meant by literals?

Answer:

The letters or alphabets that we use to represent unknowns numbers are called literals.

(ii) Define a constant.

Answer:

A symbol having a fixed value is called a constant.

(iii) What is a binomial?

Answer:

A polynomial having two terms is called a binomial.

(iv) What is an algebraic identity?

Answer:

An algebraic equation which is true for all values of the variable occurring in the relation is called an algebraic identity.

(v) Define the factorization of an algebraic expression.

Answer:

Factorization of an algebraic expression is a process by which a given algebraic expression can be expressed as a product of simpler expressions.

2. Fill in the blanks.

(i) $(a+b)^2 = \underline{a^2 + 2ab + b^2}$

(ii) (a – b)blanks-2ab + b^2

(iii) $(x + a)(x + b) = \underline{x^2 + (a + b)x + ab}$

(v) A symbol represented by a literal and can take various numerical values is called a variable.

(vi) A polynomial having only one term is called monomial.

3. Tick (✓) the correct answer.

(i) $x^2 - x = ?$

(a) x (b) ✓ x(xx- l)

(c) x^2 (d) $x - x^2$

(ii) A polynomial having two terms is called a:

(a) factorization (b) monomial

(c)✓ binomial (d) trinomial

(iii) A symbol having a fixed value is called a:

(a) term (b) variable

(c)✓ constant (d) literal

(iv) The factors of $a^2 - 9$ are:

(a)✓ $(a + 3)(a - 3)$ (b) $(a + 9)(a - 9)$

(c) $(a - 3)(a - 3)$ (d) $(a - 9)(a - 9)$

(v) $(x-y)(x-y) = ?$

(a) $x^2 - y^2$ (b) $x^2 + 2xy + y^2$

(c)✓ $x^2 - 2xy + y^2$ (d) $x^2 + y^2$

4. Resolve into factors:

Solution:

(i) $10a^2 - 200a^4b$

$= 10a^2(1 - 20a^2b)$

(ii) $36x^3y^3z^3 - 27x^2y^4z + 63xyz^4$

$= 9xyz(4x^2y^2z^2 - 3xy^3 + 7z^3)$

(iii) $15x^4y + 21x^3y^2 - 27x^2y^2 - 33xy^4$

$= 3xy(5x^3 + 7x^2y - 9xy - 11y^3)$

(iv) $xa^2 + 11x - 16a^2 - 176$

$x(a^2 + 11) - 16(a^2 + 11)$

$= (a^2 + 11)(x - 16)$

(v) $ab + abxy + ab + z^2 + x^2c + xyc + cz^2$

$= ab(x^2 + xy + z^2) + c(x^2 + xy + z^2)$

$= (x^2 + xy + z^2)(ab + c)$

$= (ab + c)(x^2 + xy + z^2)$

6. **Simplify the following polynomials.**

Solution:

(i) $(x-2y)(x+2y)$

$= x^2 - 2xy + 2xy - 4y^2 = x^2 - 4y^2$

(ii) $(4x^2)(3x+1)$

12x~3+4x~2

(iii) $2x(x+y) - 2y(x-y)$

$= 2x^2 + 2xy - 2xy + 4y^2$

$= 2x^2 + 4y^2 = 2(x^2 + 2y^2)$

(iv) $(a^2b^3)(2a-3b)$

$= 2a^3b^3 - 3a^2b^4$

(v) $(a^2 - b^2)(a^2 + b^2)$

$= a^4 - b^4$.

(vi) $(a^2+1)(a^2-a-1)$

$= a^4 - a^3 - a^2 + a^2 - a - 1$

$= a^4 - a^3 - a - 1$

(vii) $x(y+1) - y(x+1) - x - y$

$= xy + x - yx - y - x + y$

$= 0$

(viii) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$

$= 0$

7. Simplify the following by using identity.

Solution:

(i) $(3x-4)(3x+5)$

$=9x^2+15x-12x-20$

$=9x^2+3x-20$

(ii) $(2a-5b)^2$

$=(2a)^2+(5b)^2-2(2a)(5b)$

$=4a^2-20ab+25b^2$

8. Factorize.

Solution:

(i) $a^2 - 26a + 169$

$=(a)^2-2(a)(13)+(13)^2=(a-13)^2$

(ii) $1-6x^2y^2z^2+9x^4y^4z^4$

$=(1)^2-2(1)(3x^2y^2z^2)+(3x^2y^2z^2)^2$

$=(1-3x^2y^2z^2)^2$

(iii) $7ab^2-343a$

$=7a(b^2-49)=7a[(b)^2-(7)^2]$

$=7a(b-7)(b+7)$

(iv) $75-3(x-y)^2$

$=3[25-(x-y)^2]=3[(5)^2-(x-y)^2]$

$=3[(5+x-y)(5-x+y)]$

(v) $49(x+y)^2-16(x-y)^2$

$=[7(x+y)]^2-[4(x-y)]^2$

$=[7(x+y)+4(x-y)][7(x+y)-4(x-y)]$

$=(7x+7y+4x-4y)(7x+7y-4x+4y)$

$=(11x+3y)(3x+11y)$

(vi) $\frac{9}{16}a^2+ab+\frac{4}{9}b^2$

$=\left(\frac{3}{4}a\right)^2+2\left(\frac{3}{4}a\right)\left(\frac{2}{3}b\right)+\left(\frac{2}{3}b\right)^2$

$=\left(\frac{3}{4}a+\frac{2}{3}b\right)^2$

(vii) $\frac{a^2}{b^2}\ell^2-\frac{2ac}{bd}\ell m+\frac{c^2}{d^2}m^2$

$=\left(\frac{a}{b}\ell\right)^2-2\left(\frac{a}{b}\ell\right)\left(\frac{c}{d}m\right)+\left(\frac{c}{d}m\right)^2$

$=\left(\frac{a}{b}\ell-\frac{c}{d}m\right)^2$

(viii) $\left(a-\frac{9}{5}\right)^2-\frac{36}{25}m^2$

$=\left(a-\frac{9}{5}\right)^2-\left(\frac{6}{5}m\right)^2$

$=\left(a-\frac{9}{5}+\frac{6}{5}m\right)\left(a-\frac{9}{5}-\frac{6}{5}m\right)$